

Measurement of τ mass at BESIII

Kai Zhu

On behalf of BESIII

International Workshop on Physics at
Future High Intensity Collider @ 2-7GeV in China

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Outline

- Motivation
- Scan method & Data taking
- Analysis
- Summary

Motivation of high accurate τ mass measurement

Elementary parameter in SM

$$\frac{B(\tau \rightarrow e\nu_e\nu_\tau)}{\tau_\tau} = \frac{g_\tau^2 m_\tau^5}{192\pi^3}$$

Yoshio Koide equation

$$\frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}$$

Lepton universality

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau} \left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B(\tau \rightarrow e\nu_e\nu_\tau)}{B(\mu \rightarrow e\nu_e\nu_\mu)} \left(1 + \Delta_e\right) \left(1 + \Delta_W\right)$$

- $M_e = 0.510998910 \pm 0.000000013 \quad (2.6 \times 10^{-8})$
- $M_\mu = 105.658367 \pm 0.000004 \quad (3.8 \times 10^{-8})$
- $M_\tau = 1776.82 \pm 0.16 \quad (9.0 \times 10^{-5})$

PDG 2012

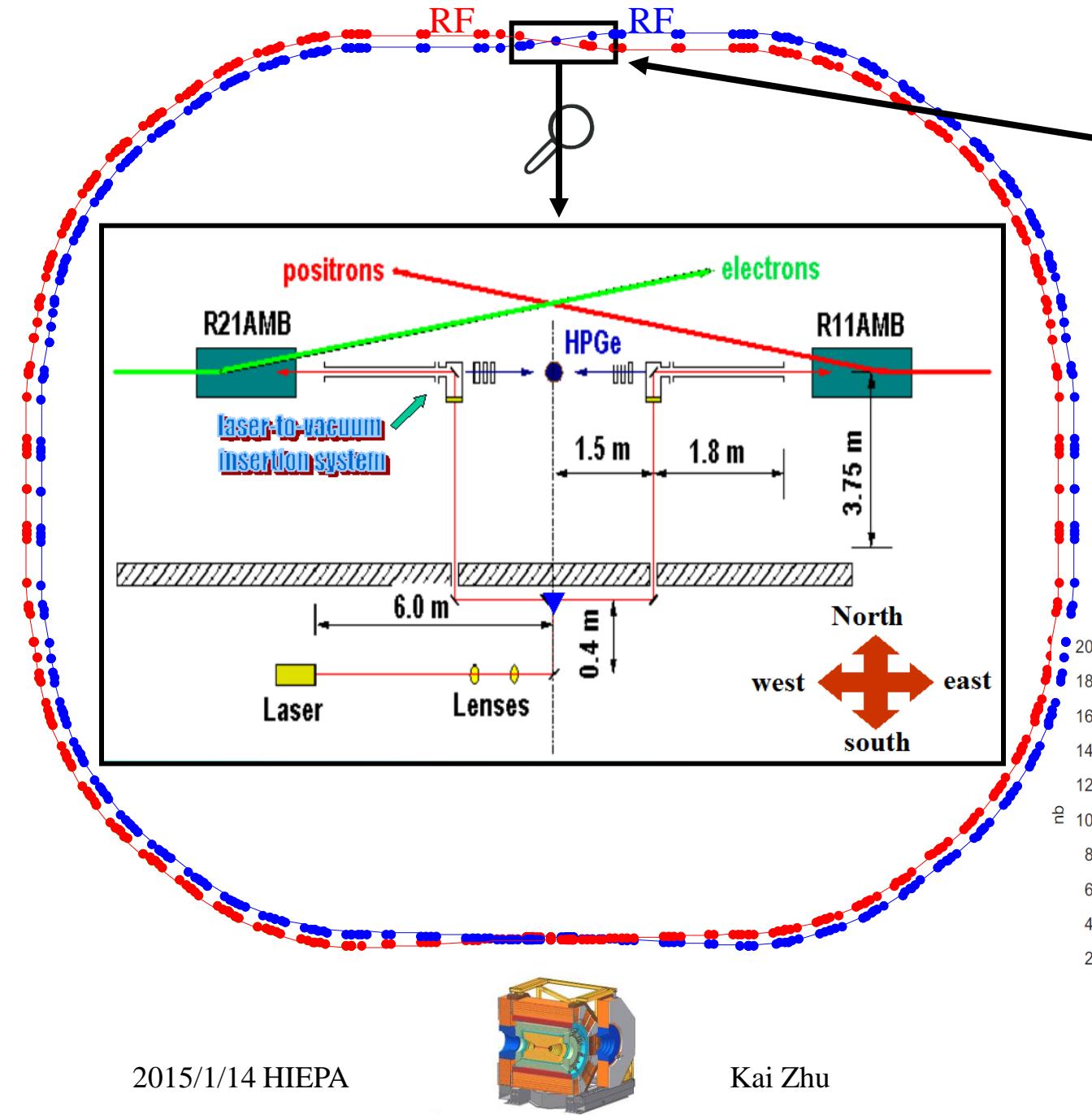
Method: Pseudo-mass and threshold scan

| τ lepton mass measurement [value+statistic +systematic error] | Year | Ex. Group | Data sample | Method |
|---|-------------|--------------|-----------------------|-------------|
| $1776.68 \pm 0.12 \pm 0.41$ | 2009 | Babar | 423 fb^{-1} | Pseudo-mass |
| $1776.81 +(+0.25 - 0.23) \pm 0.15$ | 2007 | KEDR | 6.7 pb^{-1} | Scan |
| $1776.61 \pm 0.13 \pm 0.35$ | 2007 | Belle | 414 fb^{-1} | Pseudo-mass |
| $1776.96 +(+0.18 - 0.21) +(+0.25 - 0.17)$ | 1996 | BES | 5.1 pb^{-1} | Scan |

- For **threshold scan method** precisely determine **beam energy** and **energy spread** is extremely important.

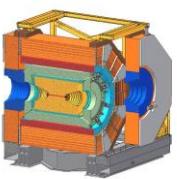
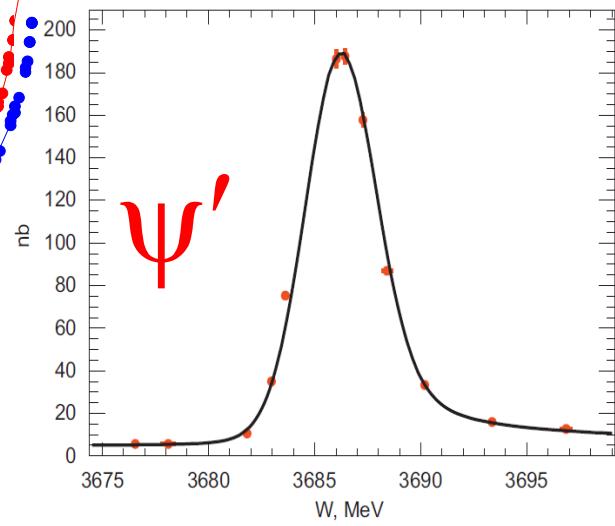
BEMS:

beam energy measurement system



$$\delta m_{\psi'}/m_{\psi'} = 2 \times 10^{-5}$$

$$\delta \Delta / \Delta = 6\%.$$



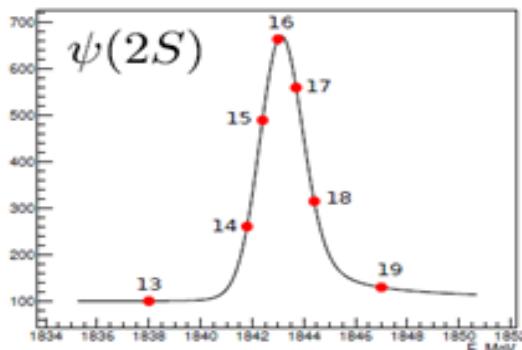
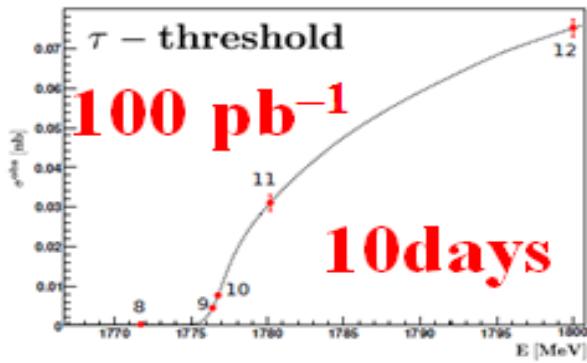
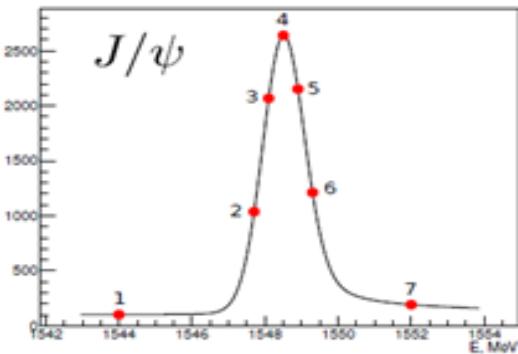
Optimization for scan

1. **N free parameters fit, N scan points is enough**
2. **The optimized position can be obtained by single parameter scan**
3. **Luminosity allocation can be determined analytically or by simulation method**

Y.K.Wang, J.Y.Zhang, X.H.Mo, C.Z.Yuan,
Chin. Phys. C 2009, 33:501-507

τ scan (plan & done)

plan



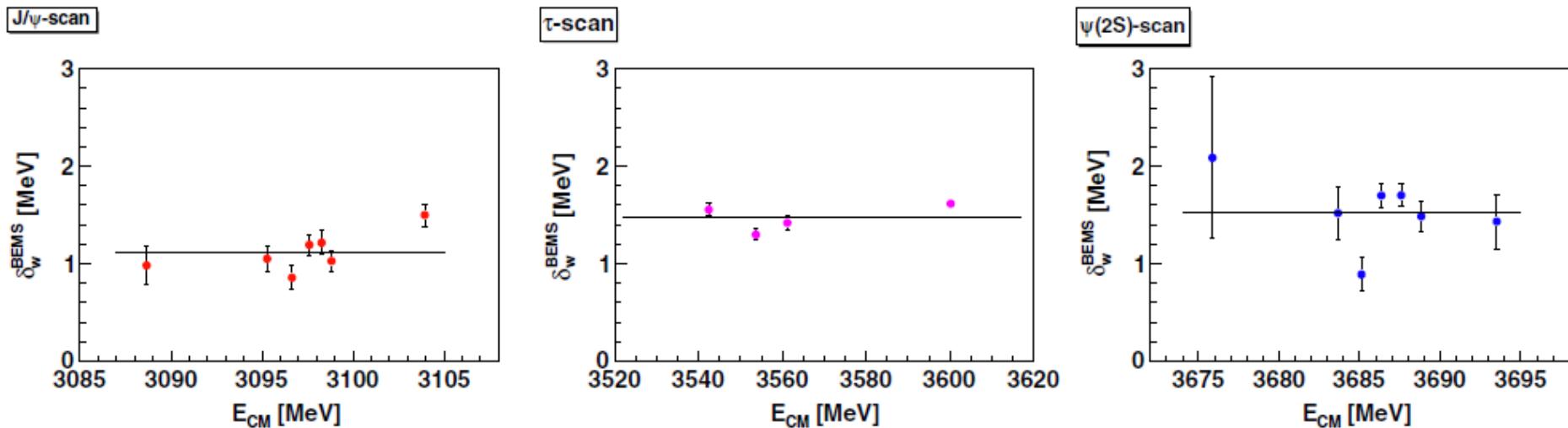
done: 24pb^{-1} four points

| Scan | $E_{\text{c.m.}}$ (MeV) | \mathcal{L} (nb ⁻¹) |
|----------|-------------------------|-----------------------------------|
| J/ψ | 3088.7 | 78.5 ± 1.9 |
| | 3095.3 | 219.3 ± 3.1 |
| | 3096.7 | 243.1 ± 3.3 |
| | 3097.6 | 206.5 ± 3.1 |
| | 3098.3 | 223.5 ± 3.2 |
| | 3098.8 | 216.9 ± 3.1 |
| | 3103.9 | 317.3 ± 3.8 |
| | 3542.4 | 4252.1 ± 18.9 |
| τ | 3553.8 | 5566.7 ± 22.8 |
| | 3561.1 | 3889.2 ± 17.9 |
| | 3600.2 | 9553.0 ± 33.8 |
| | 3675.9 | 787.0 ± 7.2 |
| ψ' | 3683.7 | 823.1 ± 7.4 |
| | 3685.1 | 832.4 ± 7.5 |
| | 3686.3 | 1184.3 ± 9.1 |
| | 3687.6 | 1660.7 ± 11.0 |
| | 3688.8 | 767.7 ± 7.2 |
| | 3693.5 | 1470.8 ± 10.3 |

Beam energy from BEMS

| Scan energy | Runs | E_{e^-} | ΔE_{e^-} | E_{e^+} | ΔE_{e^+} | E_{CM} | ΔE_{CM} |
|-------------|-------------|-----------|------------------|-----------|------------------|----------|-----------------|
| 1554.0 | 24937 | 1544.542 | 0.135 | 1544.312 | 0.217 | 3088.667 | 0.256 |
| 1547.7 | 24938-24942 | 1547.917 | 0.099 | 1547.548 | 0.106 | 3095.278 | 0.145 |
| 1548.1 | 24943-24949 | 1548.692 | 0.103 | 1548.171 | 0.086 | 3096.676 | 0.135 |
| 1548.5 | 24959-24966 | 1549.079 | 0.109 | 1548.714 | 0.075 | 3097.606 | 0.133 |
| 1548.9 | 24967-24971 | 1549.451 | 0.081 | 1549.014 | 0.114 | 3098.278 | 0.140 |
| 1549.3 | 24972-24975 | 1549.566 | 0.101 | 1549.438 | 0.083 | 3098.817 | 0.131 |
| 1552.0 | 24976-24978 | 1552.186 | 0.088 | 1551.936 | 0.107 | 3103.934 | 0.139 |
| 1771.0 | 24983-25015 | 1771.558 | 0.067 | 1771.069 | 0.053 | 3542.413 | 0.085 |
| 1777.0 | 25016-25094 | 1777.307 | 0.060 | 1776.730 | 0.046 | 3553.822 | 0.075 |
| 1780.4 | 25100-25141 | 1780.926 | 0.055 | 1780.431 | 0.065 | 3561.142 | 0.085 |
| 1800.0 | 25143-25243 | 1800.526 | 0.044 | 1799.878 | 0.044 | 3600.186 | 0.062 |
| 1838.0 | 25244-25251 | 1838.183 | 0.256 | 1837.940 | 0.157 | 3675.901 | 0.300 |
| 1841.9 | 25252-25262 | 1842.234 | 0.112 | 1841.642 | 0.281 | 3683.653 | 0.303 |
| 1842.5 | 25264-25270 | 1842.825 | 0.201 | 1842.511 | 0.112 | 3685.113 | 0.230 |
| 1843.1 | 25271-25295 | 1843.560 | 0.113 | 1843.000 | 0.152 | 3686.337 | 0.189 |
| 1843.8 | 25325-25337 | 1844.148 | 0.126 | 1843.648 | 0.095 | 3687.573 | 0.158 |
| 1844.5 | 25299-25314 | 1844.700 | 0.177 | 1844.342 | 0.140 | 3688.819 | 0.226 |
| 1847.0 | 25315-25322 | 1847.141 | 0.189 | 1846.597 | 0.156 | 3693.515 | 0.245 |

Beam energy spread from BEMS



Energy spreads (MeV)

| Scan | δ_w^{BEMS} | $\Delta(\delta_w^{\text{BEMS}})$ |
|----------|--------------------------|----------------------------------|
| J/ψ | 1.112 | 0.070 |
| τ | 1.469 | 0.064 |
| ψ' | 1.534 | 0.109 |

Data Samples and MC Simulation

- Bhabha and two gamma events → determine luminosities
 - Babayaga 3.5 generator
- Hadronic events in J/ψ and ψ' scan → study J/ψ and ψ' hadronic cross-section line shapes
 - data @ 3.097GeV, @ 3.686GeV, @ 3.650GeV used for signal /background assessment.
 - inclusive MC @ 3.097GeV and @ 3.686GeV
- 13 two-prong τ pair final states are used → determine τ mass
 - ee, e μ , e π , eK, $\mu\mu$, $\mu\pi$, μ K, π K, $\pi\pi$, KK, e ρ , $\mu\rho$ and $\pi\rho$ (with accompanying neutrinos implied)
 - KKMC + BesEvtGen

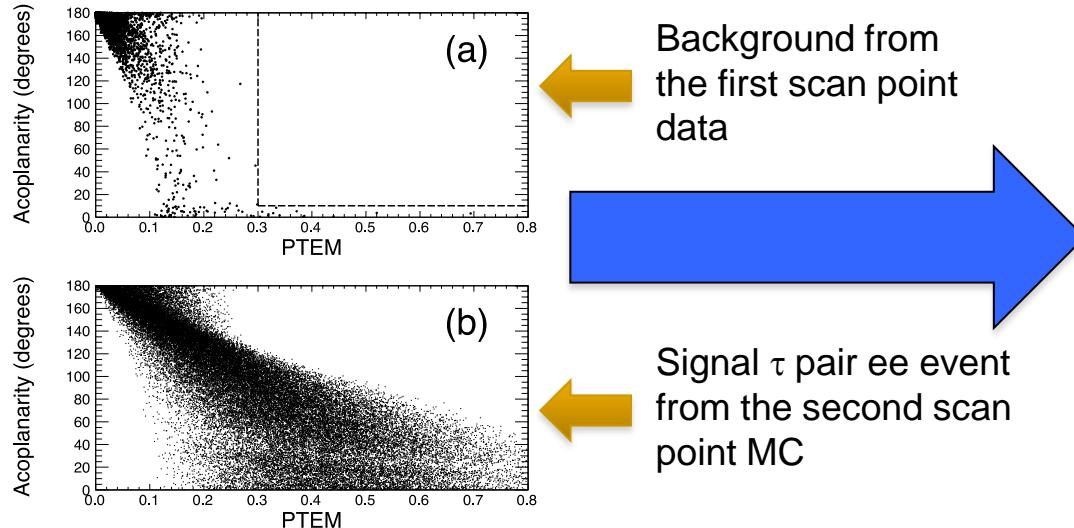
Event Selection for τ pair candidate events

- Good shower in EMC
 - $E > 25\text{MeV}$ (50MeV) for $|\cos\theta| < 0.8$ (for $0.86 < |\cos\theta| < 0.92$)
 - $0 < t < 750 \text{ ns}$, t is the time information from the EMC
- Number of good photon: 0 or 2
- Number of charged tracks: 2
- Cuts on Acoplanarity angle (θ_{acop}) and PTEM

$$PTEM = \frac{P_T}{E_{miss}^{\max}} = \frac{(c\vec{P}_1 + c\vec{P}_2)_T}{W - |c\vec{P}_1| - |c\vec{P}_2|},$$

- PID selection criteria

Event selection details

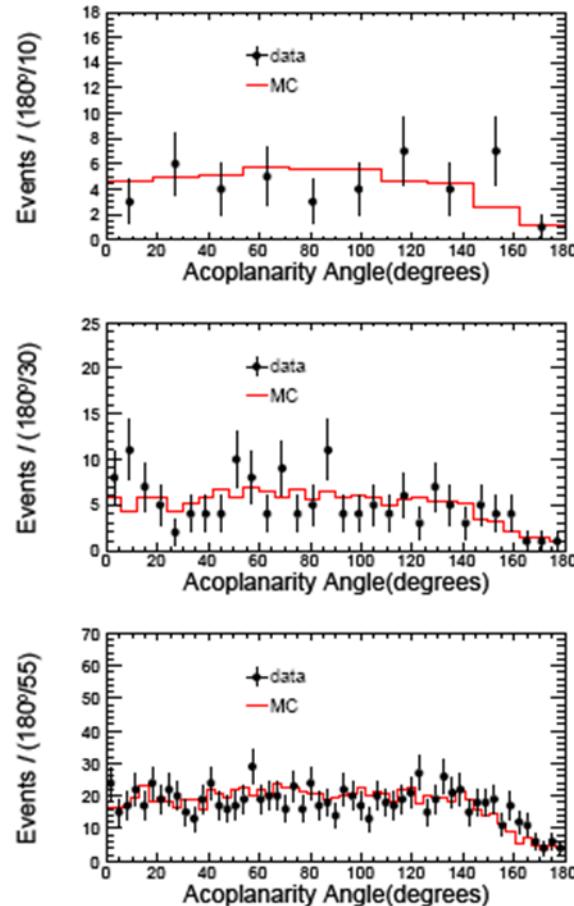


| final state | θ_{acop} | PTEM |
|-------------|-----------------|--------|
| ee | $>10^\circ$ | >0.3 |
| $e\mu$ | $<160^\circ$ | >0.1 |
| $e\pi$ | $<170^\circ$ | >0.1 |
| eK | $<170^\circ$ | |
| $\mu\mu$ | $<140^\circ$ | |
| μh | $<140^\circ$ | |
| hh | $<160^\circ$ | |
| $e\rho$ | $<170^\circ$ | |
| $\mu\rho$ | $<150^\circ$ | |
| $\pi\rho$ | | |

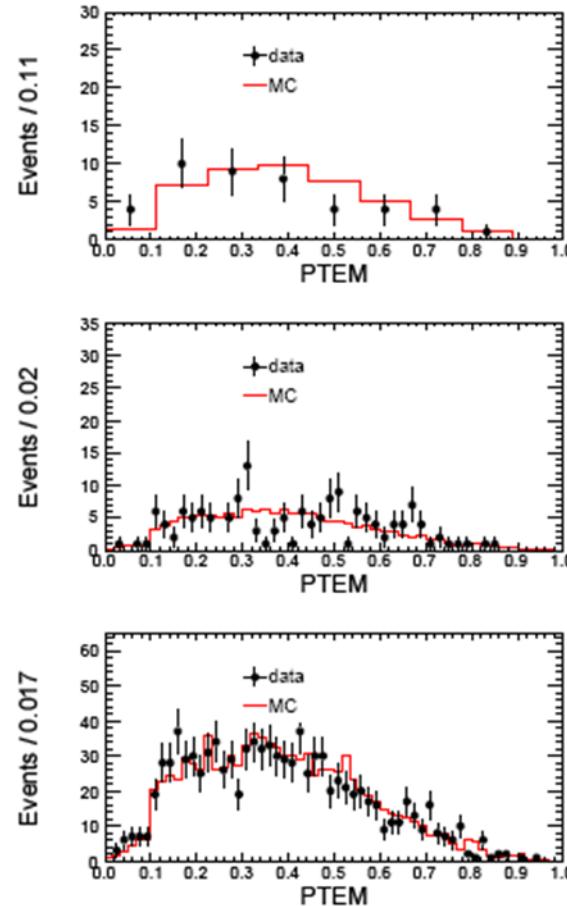
| PID | p (MeV/c) | EMC | TOF | MUC | other |
|-------|-------------------------|---------------------------------|---|--|-----------|
| e | $p_{min} < p < p_{max}$ | $0.8 < E/cp < 1.05$ | $ \Delta TOF(e) < 0.2$ ns 0 ns $< TOF < 4.5$ ns | | |
| μ | $p_{min} < p < p_{max}$ | $E/cp < 0.7$ $0.1 < E < 0.3$ | $ \Delta TOF(\mu) < 0.2$ ns | $(D > (80 \times p - 50) \text{ cm or } D > 40 \text{ cm})$ and $N_h > 1$ | |
| π | $p_{min} < p < p_{max}$ | $E/cp < 0.6$ | $ \Delta TOF(\pi) < 0.2$ ns 0 ns $< TOF < 4.5$ ns | | not μ |
| K | $p_{min} < p < p_{max}$ | $E/cp < 0.6$ | $ \Delta TOF(K) < 0.2$ ns 0 ns $< TOF < 4.5$ ns | | not μ |

Comparison of the data and MC Samples

Acoplanarity angle



PTEM



1st – point below threshold

2nd – point

$E_{cm} = 3553.8\text{MeV}$

3rd – point

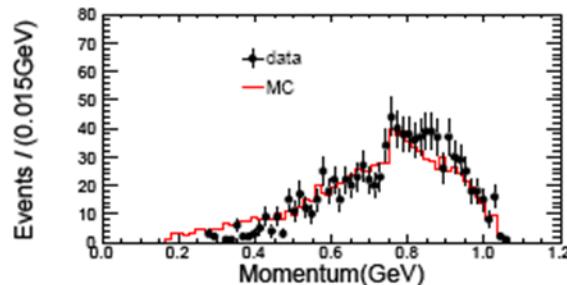
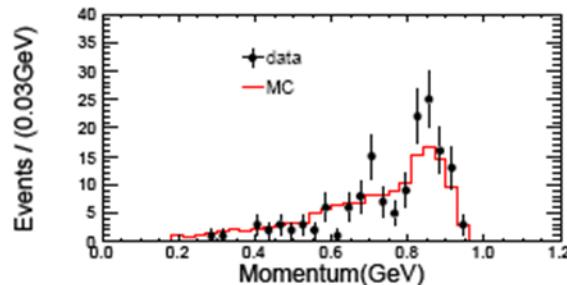
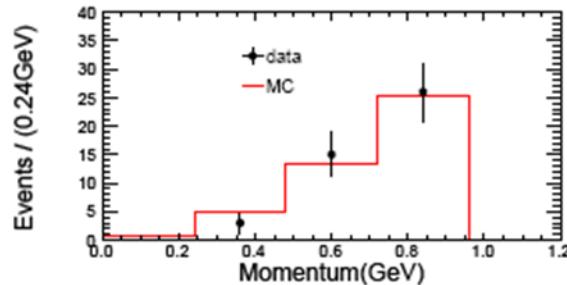
$E_{cm} = 3561.1\text{MeV}$

4th – point

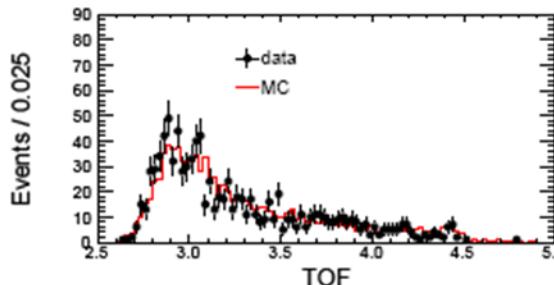
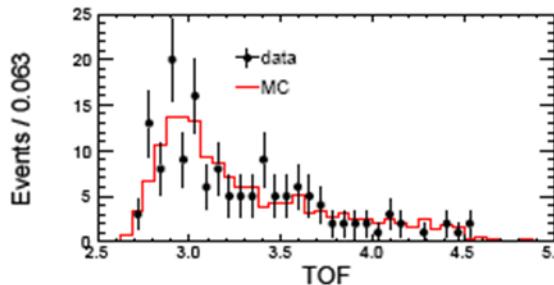
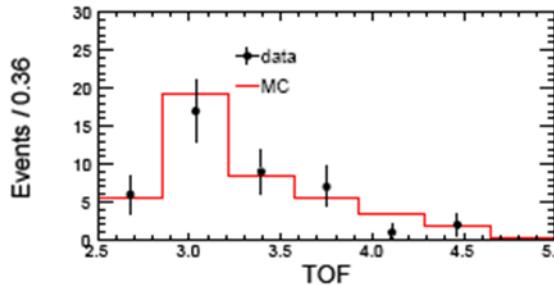
$E_{cm} = 3600.2\text{MeV}$

Comparison of the data and MC Samples (Cont.)

Momentum of charged tracks



TOF of charged tracks
1st – point below threshold



2nd – point
 $E_{cm} = 3553.8\text{MeV}$

3rd – point
 $E_{cm} = 3561.1\text{MeV}$

4th – point
 $E_{cm} = 3600.2\text{MeV}$

Data analysis

- Luminosity
- J/ψ and ψ' line shape (calibration of energy scale and spread)
- τ mass measurement

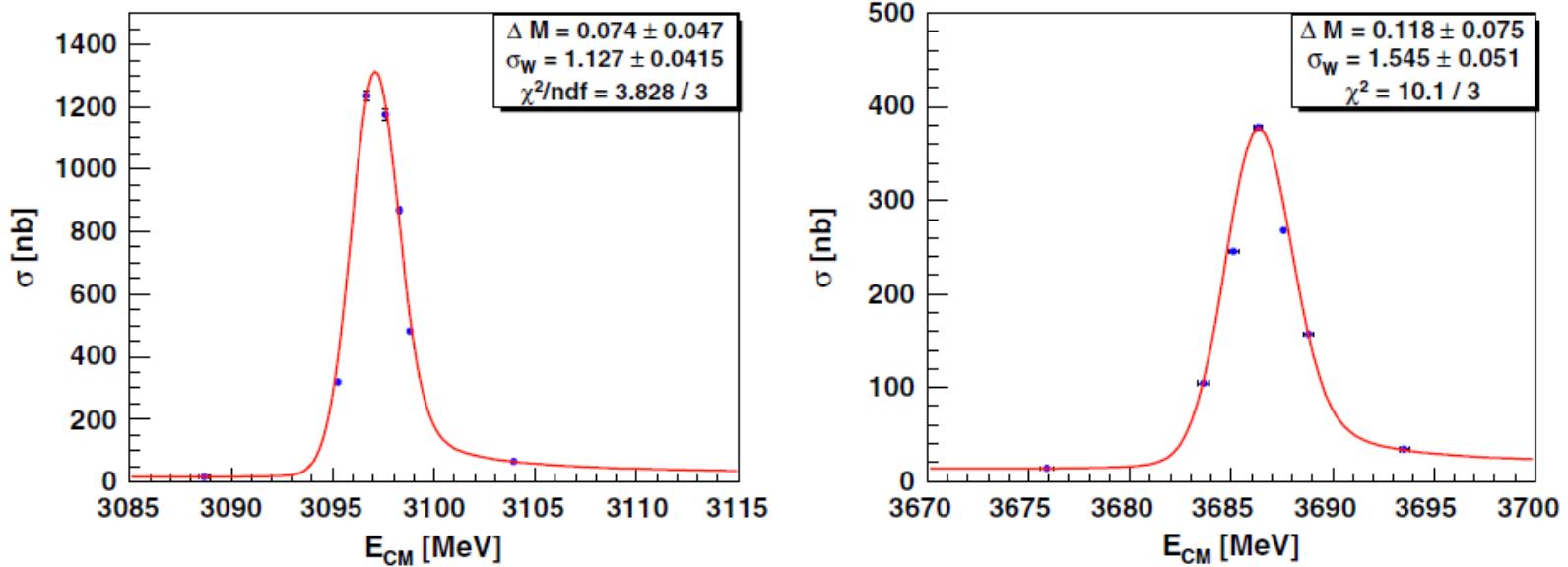
Luminosity

Cross checked by Bhabha and $\gamma\gamma$

| Scan | E_{CMS} (GeV) | \mathcal{L}_{Bhabha} (nb $^{-1}$) | $\mathcal{L}_{\gamma\gamma}$ (nb $^{-1}$) | $\mathcal{L}_{Bhabha}/\mathcal{L}_{\gamma\gamma}$ |
|----------|--------------------|---|---|---|
| J/ψ | 3.0887 | 77.95 ± 0.81 | 78.5 ± 1.9 | 0.993 ± 0.026 |
| | 3.0953 | 223.6 ± 2.6 | 219.3 ± 3.1 | 1.020 ± 0.019 |
| | 3.0967 | 247.4 ± 2.1 | 243.1 ± 3.3 | 1.018 ± 0.016 |
| | 3.0976 | 202.6 ± 1.8 | 206.5 ± 3.1 | 0.981 ± 0.017 |
| | 3.0983 | 223.2 ± 2.2 | 223.5 ± 3.2 | 0.999 ± 0.017 |
| | 3.0988 | 213.9 ± 2.2 | 216.9 ± 3.1 | 0.986 ± 0.018 |
| | 3.1039 | 312.9 ± 2.4 | 317.3 ± 3.8 | 0.986 ± 0.014 |
| τ | 3.5424 | 4283.4 ± 26.5 | 4252.1 ± 18.9 | 1.007 ± 0.008 |
| | 3.5538 | 5595.9 ± 34.4 | 5566.7 ± 22.8 | 1.005 ± 0.007 |
| | 3.5611 | 3873.0 ± 24.0 | 3889.2 ± 17.9 | 0.996 ± 0.008 |
| | 3.6002 | 9581.3 ± 58.5 | 9553.0 ± 33.8 | 1.003 ± 0.007 |
| ψ' | 3.6759 | 788.2 ± 5.5 | 787.0 ± 7.2 | 1.001 ± 0.012 |
| | 3.6837 | 835.4 ± 6.2 | 823.1 ± 7.4 | 1.015 ± 0.012 |
| | 3.6851 | 836.7 ± 6.0 | 832.4 ± 7.5 | 1.005 ± 0.012 |
| | 3.6863 | 1209.4 ± 8.0 | 1184.3 ± 9.1 | 1.021 ± 0.010 |
| | 3.6876 | 1672.8 ± 11.1 | 1660.7 ± 11.0 | 1.007 ± 0.009 |
| | 3.6888 | 788.7 ± 5.6 | 767.7 ± 7.2 | 1.027 ± 0.012 |
| | 3.6935 | 1497.3 ± 9.8 | 1470.8 ± 10.3 | 1.018 ± 0.010 |

Most
 $\delta L/L < 2\%$

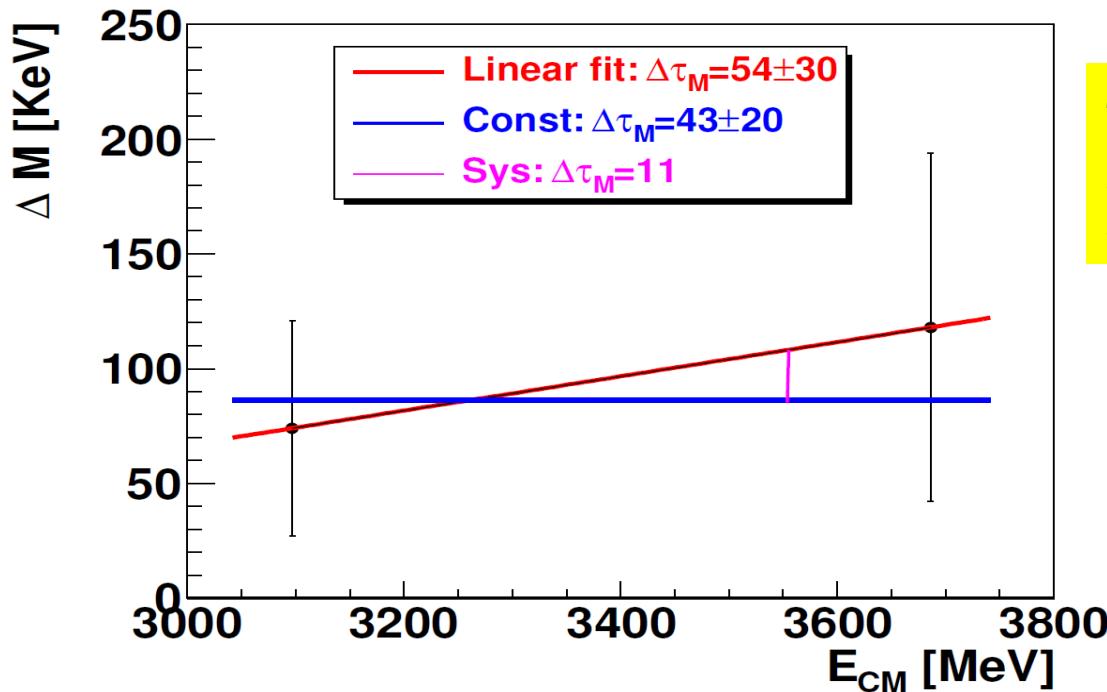
J/ψ and ψ' line shape fit



$$\Delta M = M_{FIT} - M_{PDG} \quad \delta_w = \text{energy spread}$$

| Scan | ΔM | δ_w | |
|----------|-----------------------------|-----------------------------|-----|
| J/ψ | $0.074 \pm 0.047 \pm 0.043$ | $1.127 \pm 0.042 \pm 0.050$ | MeV |
| ψ' | $0.118 \pm 0.076 \pm 0.021$ | $1.545 \pm 0.051 \pm 0.069$ | |

Energy scale extrapolation to τ mass region



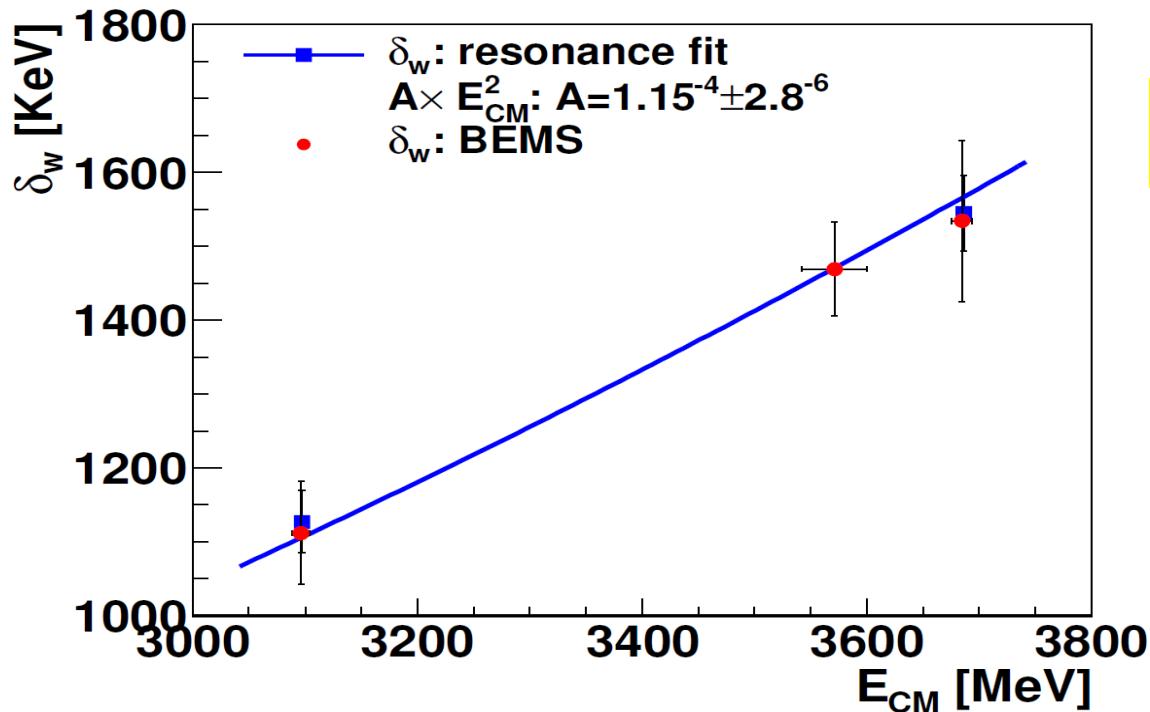
Two assumptions:
linear and constant

$\Delta M \approx 90$ keV

$\Delta M \Rightarrow \delta M_\tau$

$$\delta M_\tau = 54 \pm 30 \text{ (stat.)} \pm 12 \text{ (sys.)} \text{ keV}$$

Energy spread extrapolation to τ mass region



$$\Delta\delta_w \Rightarrow \delta M_\tau$$

$$\delta M_\tau = 16 \text{ keV}$$

Quadratic dependence

$$\delta_w = 1.469 \pm 0.064 \\ \pm 0.057 \text{ MeV}$$



Interference is the main contribution to the systematic

The number of observed events and that of normalized MC samples

| Final state | 1 | | 2 | | 3 | | 4 | | Total | |
|-------------|------|----|------|------|------|-------|------|-------|-------|--------|
| | Data | MC | Data | MC | Data | MC | Data | MC | Data | MC |
| ee | 0 | 0 | 4 | 3.7 | 13 | 12.2 | 84 | 76.1 | 101 | 92.0 |
| $e\mu$ | 0 | 0 | 8 | 9.1 | 35 | 31.4 | 168 | 192.6 | 211 | 233.1 |
| $e\pi$ | 0 | 0 | 8 | 8.6 | 33 | 29.7 | 202 | 184.4 | 243 | 222.6 |
| eK | 0 | 0 | 0 | 0.5 | 2 | 1.8 | 16 | 16.9 | 18 | 19.3 |
| $\mu\mu$ | 0 | 0 | 2 | 2.9 | 8 | 9.2 | 49 | 56.3 | 59 | 68.4 |
| $\mu\pi$ | 0 | 0 | 4 | 3.9 | 11 | 14.1 | 89 | 86.7 | 104 | 104.7 |
| μK | 0 | 0 | 0 | 0.2 | 3 | 0.8 | 7 | 9.0 | 10 | 10.1 |
| $\pi\pi$ | 0 | 0 | 1 | 2.0 | 5 | 7.7 | 57 | 54.0 | 63 | 63.8 |
| πK | 0 | 0 | 1 | 0.3 | 0 | 0.8 | 10 | 8.2 | 11 | 9.3 |
| KK | 0 | 0 | 0 | 0.0 | 1 | 0.1 | 1 | 0.3 | 2 | 0.4 |
| $e\rho$ | 0 | 0 | 3 | 6.1 | 19 | 20.6 | 142 | 132.0 | 164 | 158.7 |
| $\mu\rho$ | 0 | 0 | 8 | 3.3 | 8 | 11.8 | 52 | 63.3 | 68 | 78.5 |
| $\pi\rho$ | 0 | 0 | 5 | 3.4 | 15 | 10.8 | 97 | 96.0 | 117 | 110.2 |
| Total | 0 | 0 | 44 | 44.2 | 153 | 151.2 | 974 | 975.7 | 1171 | 1171.0 |

Agree well

Formulae prepared for fit

$$L(m_\tau, \mathcal{R}_{\text{data/MC}}, \sigma_B) = \prod_{i=1}^4 \frac{\mu_i^{N_i} e^{-\mu_i}}{N_i!} \quad m_\tau, R_{\text{data/MC}}, \sigma_B \text{ are floating}$$

$$\mu_i = [\mathcal{R}_{\text{data/MC}} \times \epsilon_i \times \sigma(E_{\text{c.m.}}^i, m_\tau) + \sigma_B] \times \mathcal{L}_i$$

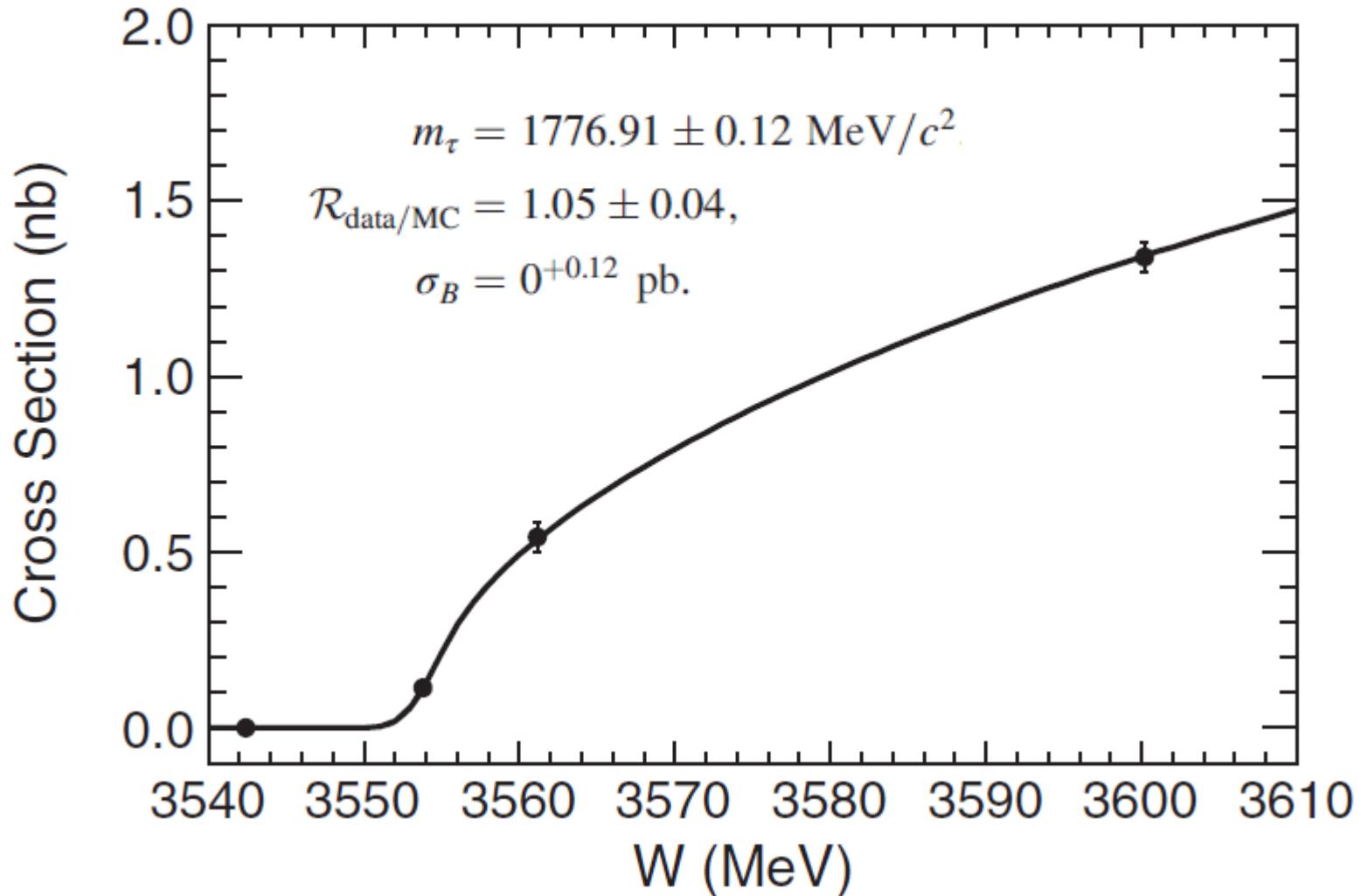
$$\sigma(E_{\text{c.m.}}, m_\tau, \delta_w^{\text{BEMS}}) = \frac{1}{\sqrt{2\pi}\delta_w^{\text{BEMS}}} \int_{2m_\tau}^{\infty} dE'_{\text{c.m.}} e^{\frac{-(E_{\text{c.m.}} - E'_{\text{c.m.}})^2}{2(\delta_w^{\text{BEMS}})^2}} \int_0^{1 - \frac{4m_\tau^2}{E'^2_{\text{c.m.}}}} dx F(x, E'_{\text{c.m.}}) \frac{\sigma_1(E'_{\text{c.m.}} \sqrt{1-x}, m_\tau)}{|1 - \prod(E_{\text{c.m.}})|^2}$$

Theoretical accuracy of cross section at the level of **0.1%**

$F(x, E_{\text{c.m.}})$: E.A.Kuraev,V.S.Fadin , Sov.J.Nucl.Phys. 41(1985)466;

$\Pi(E_{\text{c.m.}})$: F.A. Berends et al. , Nucl. Phys. B57 (1973)381.

Fit results



Systematic uncertainties

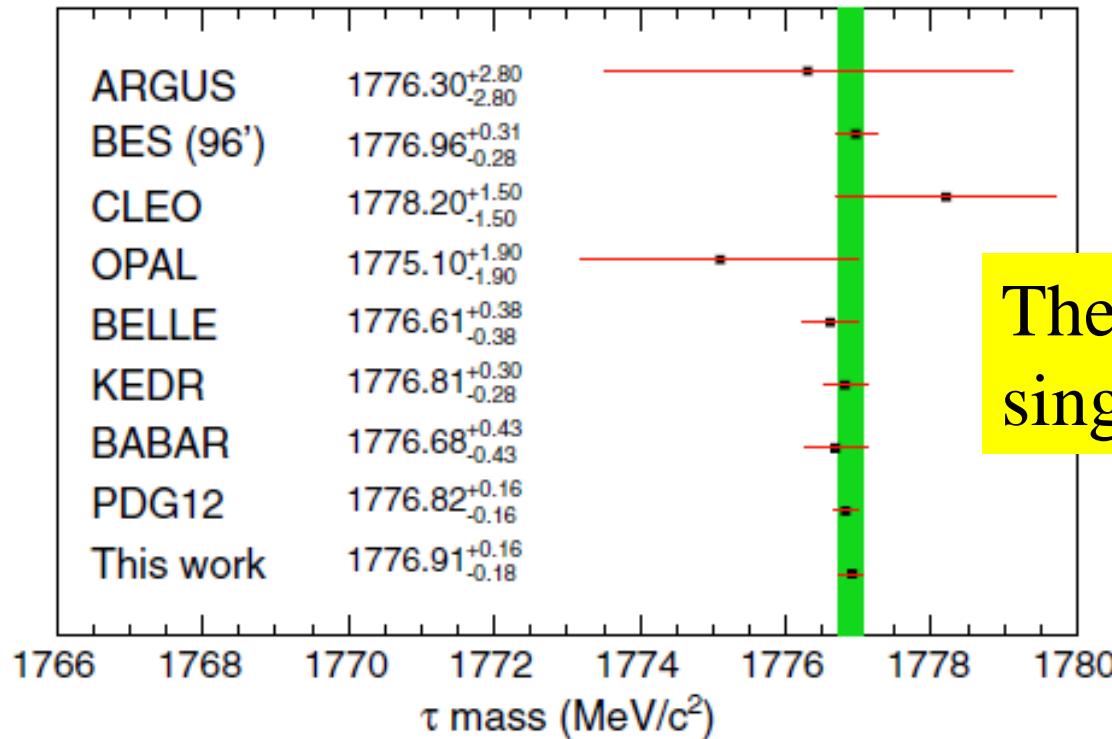
| Source | Δm_τ (MeV/c ²) |
|-------------------------------------|---------------------------------------|
| Theoretical accuracy | 0.010 |
| Energy scale | +0.022 -0.086 |
| Energy spread | 0.016 |
| Luminosity | 0.006 |
| Cut on number of good photons | 0.002 |
| Cuts on PTEM and acoplanarity angle | 0.05 |
| mis-ID efficiency | 0.048 |
| Background shape | 0.04 |
| Fitted efficiency parameter | +0.038 -0.034 |
| Total | +0.094 -0.124 |

Summary

BESIII

$$m_\tau = (1776.91 \pm 0.12^{+0.10}_{-0.13}) \text{ MeV}/c^2$$

PRD 90, 012001



The most precise
single measurement.

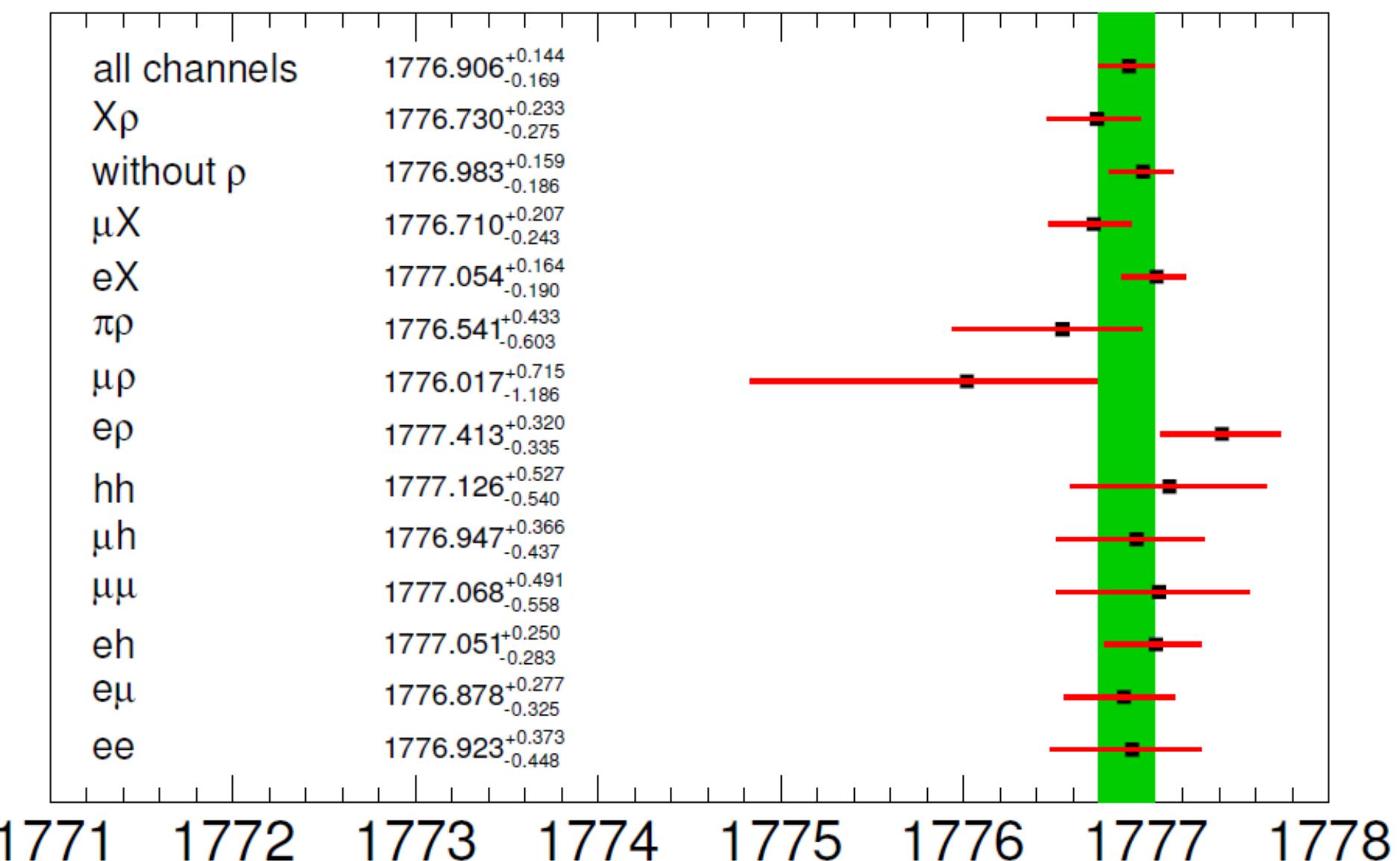
$$\left(\frac{g_\tau}{g_\mu}\right)^2 = 1.0016 \pm 0.0042$$

Lepton universality can be tested at 0.4%

- An updated scan may at BESIII
 - 10 days → 29 days, $24\ pb^{-1}$ → $100\ pb^{-1}$

Thanks!

Backup



Tau mass scan is actually an accelerator experiment

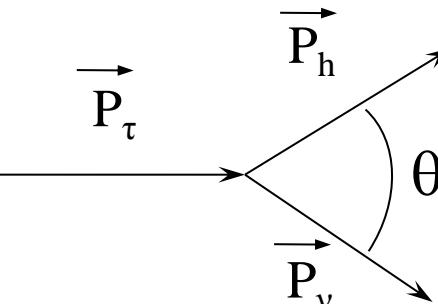
Tau mass scan

Expected luminosity (speed of data accumulation) is 0.3 pb-1/hour. Expected time of hitting to energy point is 1 day (3 steps x 4+4 measurements of electron/positron beam energy). Then it will take about 29 days to do experiment.

| Beam energy, Mev | 1771.0 | 1776.6 | 1777.0 | 1780.4 | 1800.0 | Psi prime scan | 1776.6 | 1777.0 | J/psi scan |
|-------------------------------|--------|--------|--------|--------|--------|----------------|--------|--------|------------|
| Integrated luminosity, pb-1 | 14 | 14 | 14 | 7 | 14 | | 25 | 12 | |
| Time of data aquisition, days | 3 | 3 | 3 | 2 | 3 | 4 | 4,5 | 2,5 | 4 |

Old plan 10 days → New plan: 29 days

Pseudomass method



All in CMS

$$M_{\min} \leq M_\tau$$

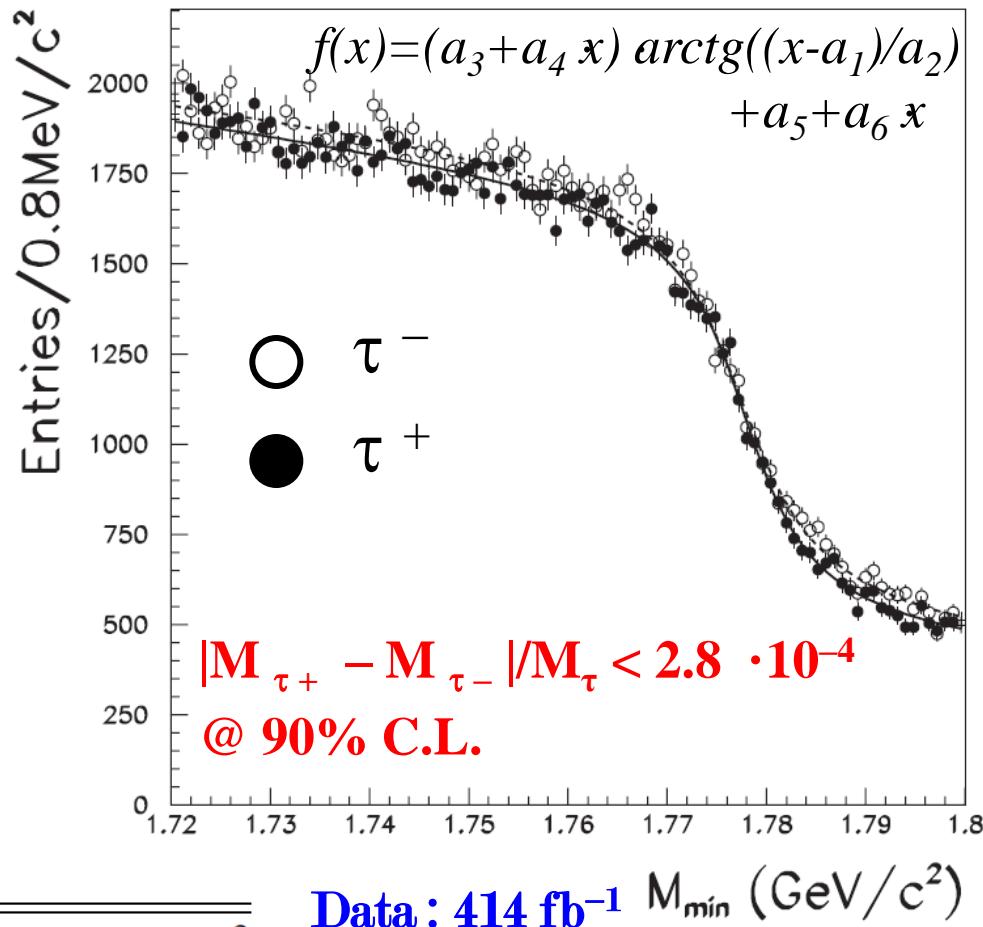
$$M_{\min}^2 = M_h^2 + 2(E_\tau - E_h)(E_h - P_h)$$

$E_\tau = E_{\text{beam}}$: beam energy, run dependence is corrected

E_h : hadron system energy

P_h : hadron system momentum

M_h : mass of the hadron system

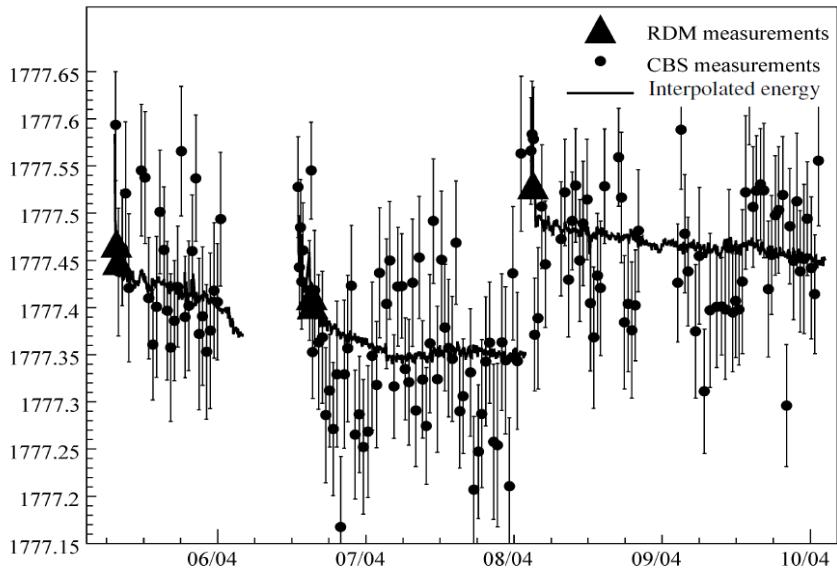


Data: 414 fb^{-1} M_{\min} (GeV/c^2)

$M_\tau = 1776.61 \pm 0.13(\text{stat.}) \pm 0.35(\text{sys.}) \text{ MeV}$
(Belle:PRL99,011801)

Threshold scan method

VEPP-4M energy (MeV)



8 points, 6.7 pb^{-1} , for τ
1 points, 0.8 pb^{-1} , at ψ'

Beam energy determination

Detection efficiency variations

Energy spread determination accuracy

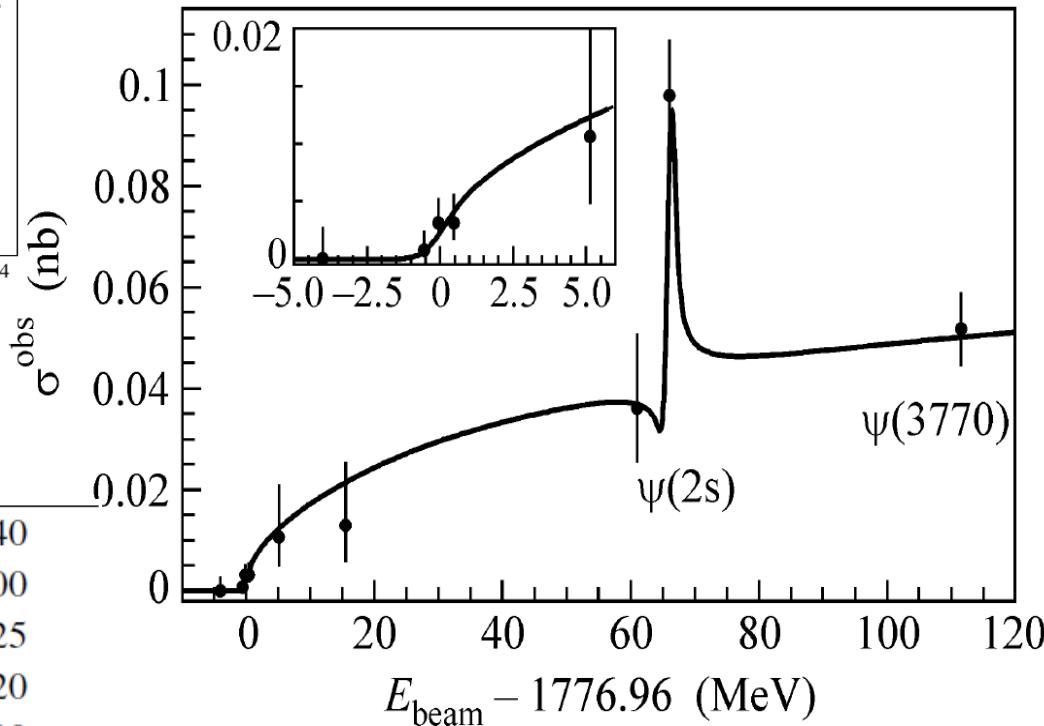
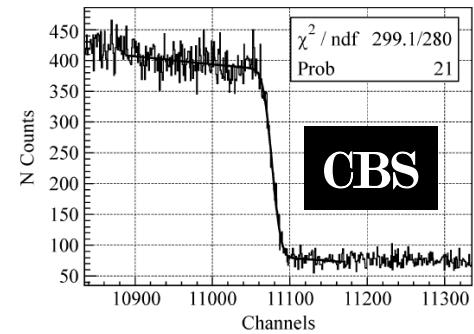
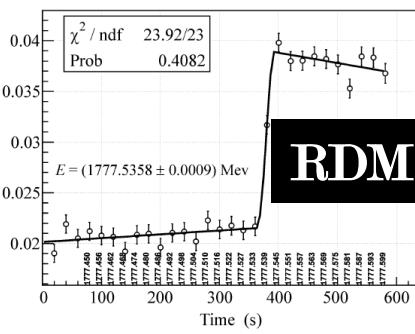
Energy dependence of the background

Luminosity measurement instability

Beam energy spread variation

Cross section calculation (r.c., interference)

Sum in quadrature



$$M_\tau = 1776.81^{+0.25}_{-0.23} \pm 0.15 \text{ MeV}$$

$$\sigma M_\tau / M_\tau = 1.64 \times 10^{-4}$$

KDER: JETPL85_347

CM energy setting

$$E_{cm}^{AA} = (E_{e^+} + E_{e^-}) \cdot \cos \frac{\alpha}{2}$$

$$E_{cm}^{GA} = 2\sqrt{E_{e^+}E_{e^-}} \cdot \cos \frac{\alpha}{2}$$

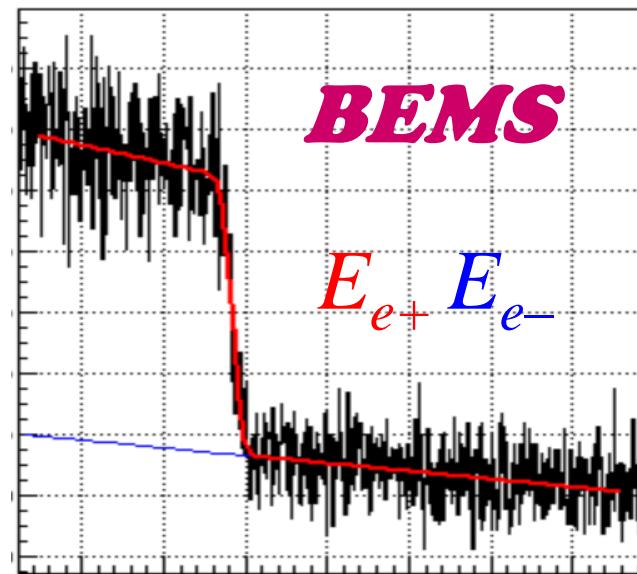
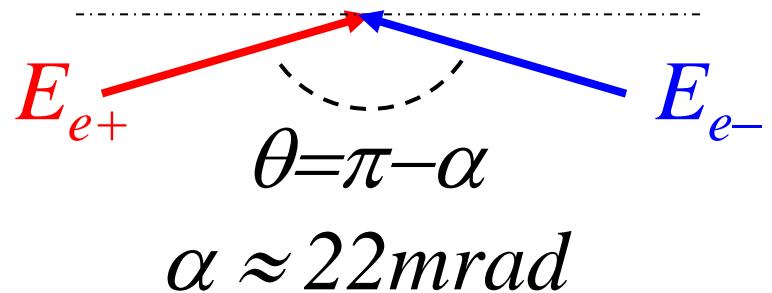
$$E_{cm}^{GA} \approx E_{cm}^{AA} \approx 2E_{beam} \left(1 - \frac{\alpha^2}{8} \right)$$

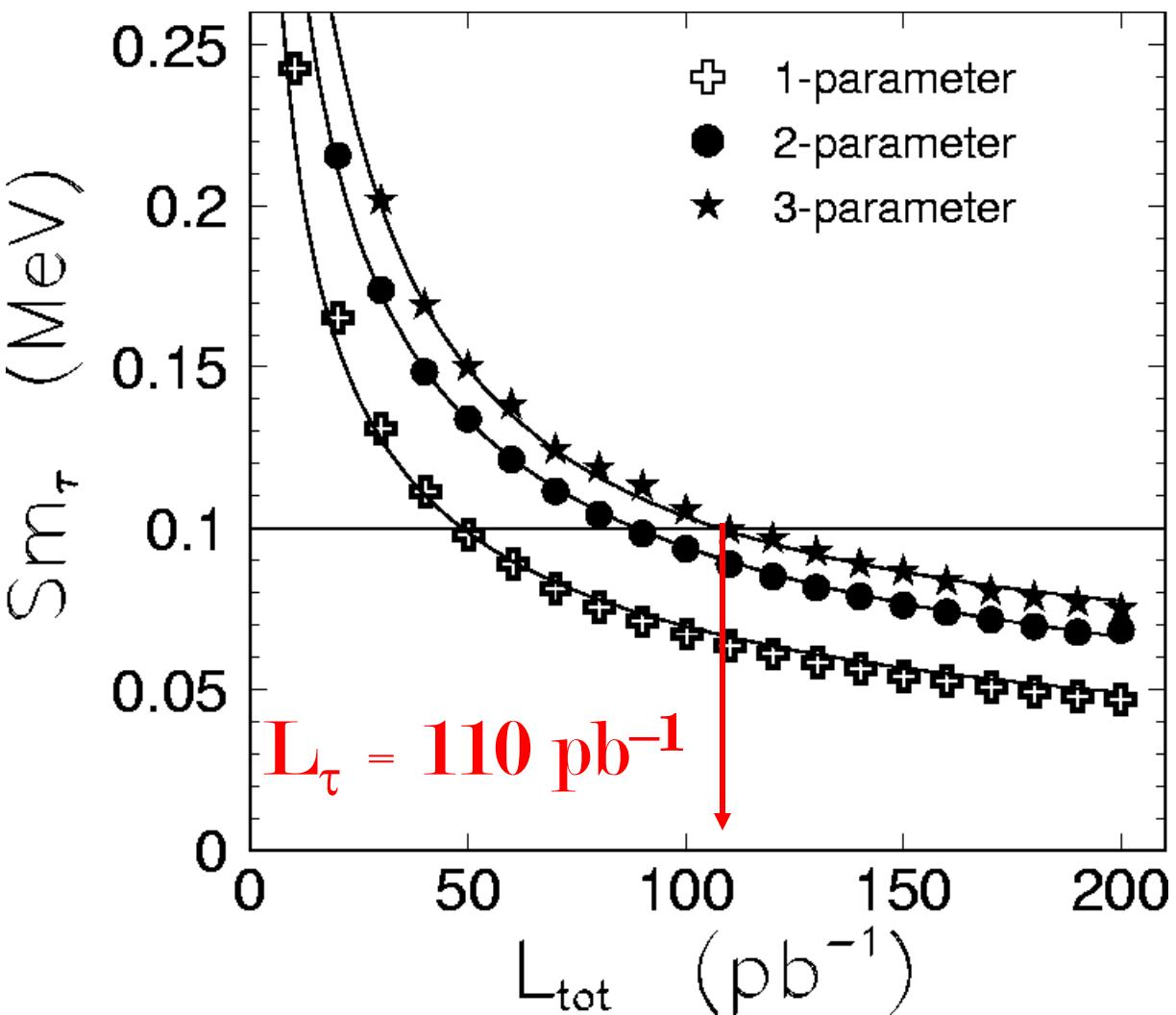
$E_{cm}^{GA} \approx E_{cm}^{AA} \approx E_{cm} \text{ vs}$
 $\alpha\text{-effect} \approx 6 \times 10^{-5},$
 $0.11 \text{ MeV} @ \tau \text{ threshold}$

$$E_{cm}^{\sqrt{S}} \approx 2E_{beam} \left(1 - \frac{\alpha^2}{8} \right)$$

$$E_{cm}^{\sqrt{S}} = \sqrt{2m_e^2 + 2E_{e^+}E_{e^-} - 2\sqrt{E_{e^+}^2 - m_e^2}\sqrt{E_{e^-}^2 - m_e^2} \cdot \cos(\pi - \alpha)}$$

$$S = (E_{e^+} + E_{e^-})^2 - (p_{e^+} + p_{e^-})^2$$





Optimization study:
 Chin. Phys. C 2009,
 33:501-507 ;
 Y.K.Wang,
 J.Y.Zhang,
 X.H.Mo,
 C.Z.Yuan.

Only based on
 $e\mu$ event !!
 Only Statistics
 uncertainty !!

$$1 = M_\tau, 2 = \epsilon, 3 = \sigma_{\text{BG}};$$

$$L_1 : L_2 = 3:1, L_1 : L_{\text{tot}} = 10\%, \delta M_\tau \propto (\sqrt{L})^{-1};$$

Theoretical accuracy of cross section at the level of 0.1%

$$\sigma(W) = \frac{1}{\sqrt{2\pi}\Delta_E} \int_0^{+\infty} dW' e^{-(W'-W)^2/2\Delta_E^2} \int_0^{\beta^2} dx F_i(x, W') \sigma^0(W' \sqrt{1-x})$$

Diagram illustrating the components of the theoretical cross section:

- Energy Spread**: Points to the term Δ_E in the denominator.
- ISR correction**: Points to the term $F_i(x, W')$.
- Coulomb Correction**: Points to the term $\sigma^0(W)$.
- FSR Correction**: Points to the term $\sigma^0(W)$.
- Vacuum Polarization Correction**: Points to the term $\sigma^0(W)$.

$$\sigma^0(W) = \frac{4\pi\alpha^2}{3W^2} \frac{\beta(3-\beta^2)}{2} \frac{F_c(\beta)F_r(\beta)}{\left[1 - \Pi(W)\right]^2}$$