

# Charmed Meson Leptonic and Hadronic decays at BESIII

**BESIII**

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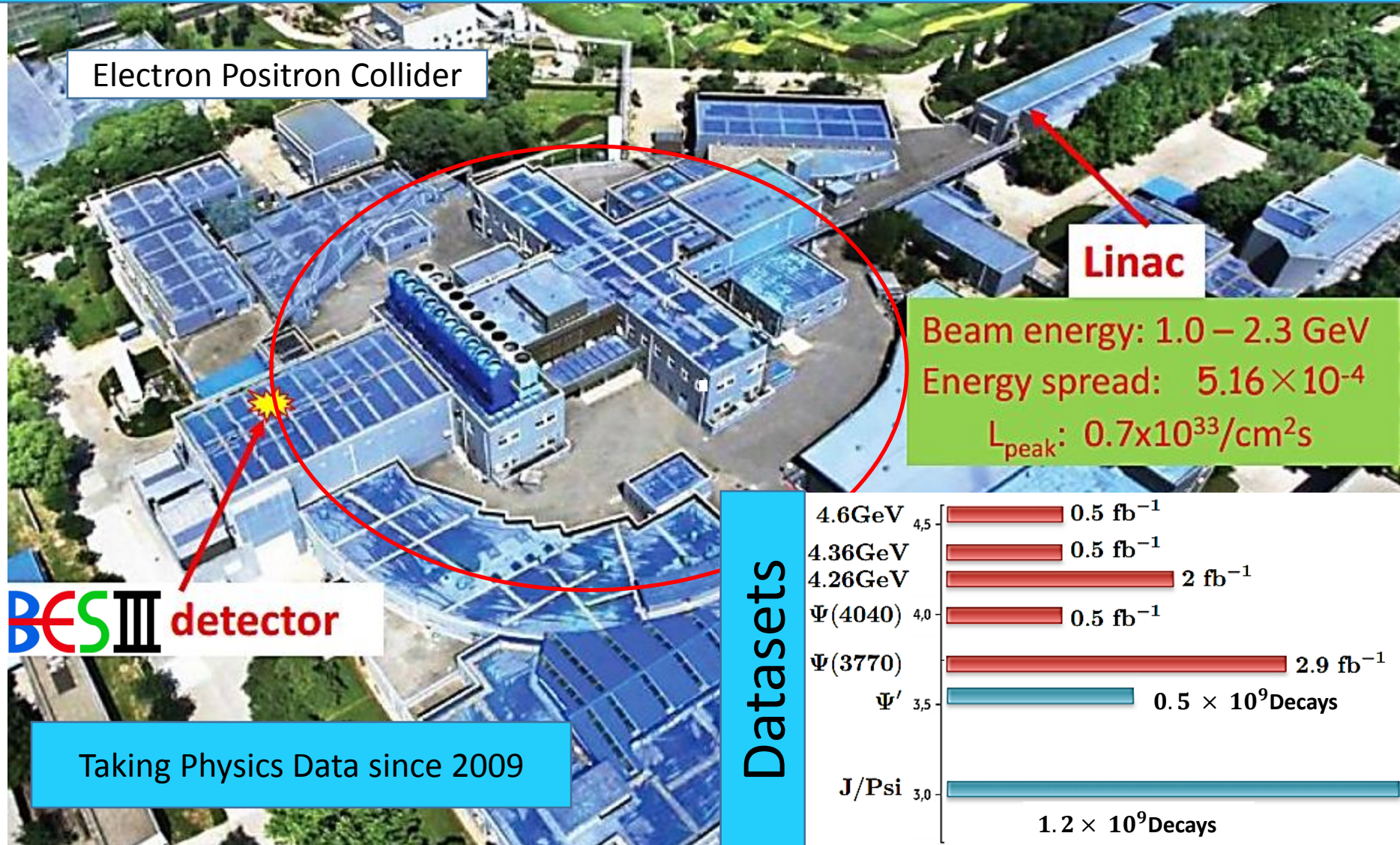
CIPANP 2015

May 24, 2015

# Overview

- BESIII experiment
- Leptonic decays
  - $D^+ \rightarrow \mu \nu$
- Semi-leptonic decays
  - $D^0 \rightarrow K(\pi)^- e^+ \nu$
  - Measurement of  $Y_{\text{CP}}$  in  $D^0 \bar{D}^0$  oscillation
- Hadronic decays
  - GGSZ strong-phase measurements for  $D^0 \rightarrow K^0 \pi^+ \pi^-$
- More upcoming BESIII measurements
- Summary

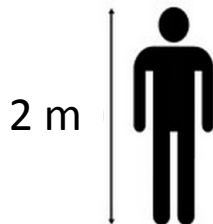
# BEPCII and BESIII



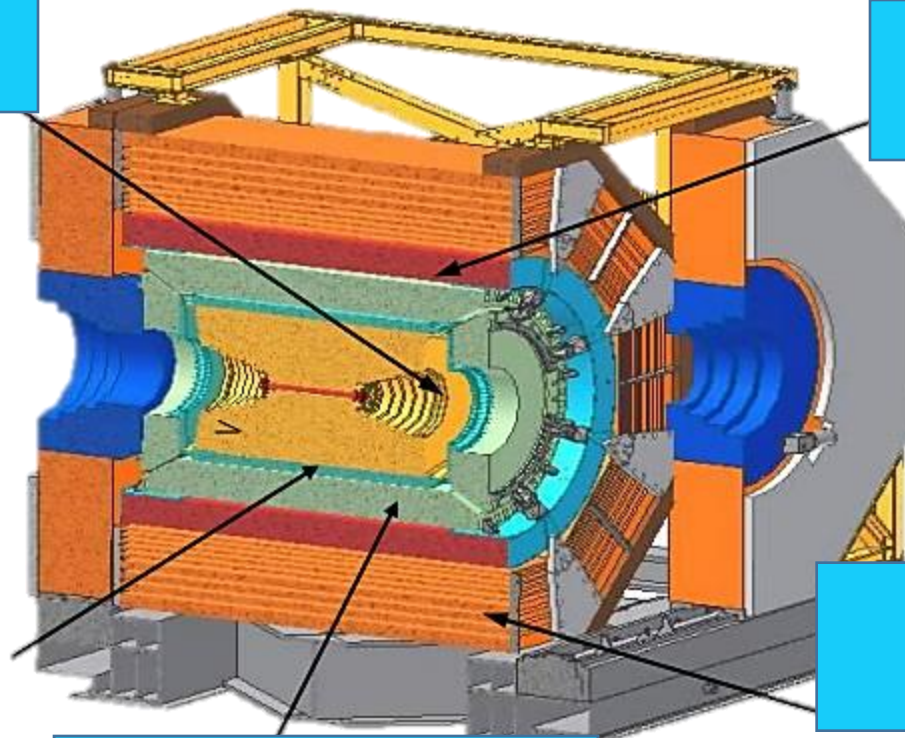
# BESIII Detector

Drift Chamber (MDC)  
 $\sigma_P/P = 0.5\% @1 \text{ GeV}$   
 $\sigma_{dE/dx} = 6\%$

Super-conducting  
magnet (1.0 tesla)



Time Of Flight (TOF)  
 $\sigma_T : 90 \text{ ps}$  Barrel  
 $110 \text{ ps}$  endcap



$\mu$  Counter  
8- 9 layers RPC

EMC :  
 $\sigma_E/E = 2.5\% @1 \text{ GeV}$   
 $\sigma_Z = 0.6 \text{ cm}$

M. Ablikim et al., (BESIII Collaboration),  
Nucl. Instrum. Meth. A 614, 345 (2010).

53 institutions, more than 400 physicists



## USA

Carnegie Mellon University, Indiana University, University of Hawaii, University of Minnesota, University of Rochester.



## Germany

Bochum University, GSI Darmstadt, Helmholtz Institute Mainz, Johannes Gutenberg University of Mainz, Universitaet Giessen



## Sweden

Uppsala University



## Nederland

KVII University of Groningen



## Italy

Ferrara University, Laboratori Nazionali di Frascati, University of Turin, Perugia



## Turkey

Turkish Accelerator Center Particle Factory Group



## Pakistan

Institute of Information Technology, University of the Punjab.



## Russia

Budker Institute of Nuclear Physics, Joint Institute for Nuclear Research



## Japan

Tokyo University



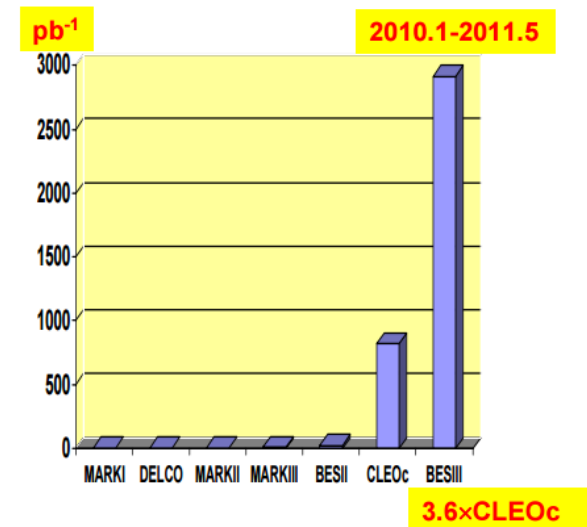
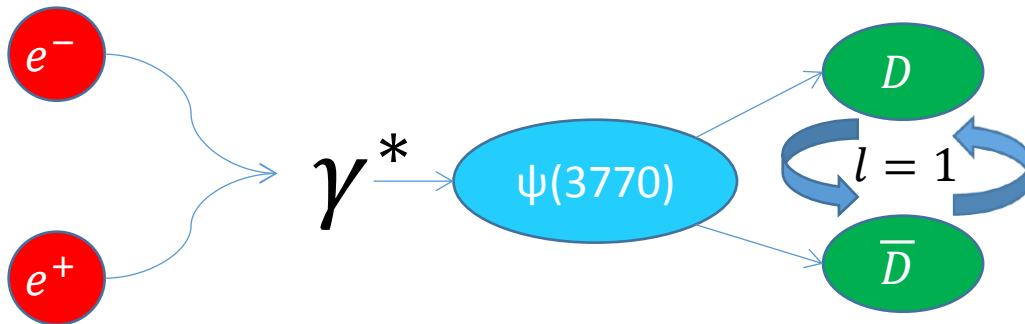
## China

31 institutions

# $\psi(3770)$ Dataset

$2.9 \text{ fb}^{-1}$  is the largest set of at this type in the world by 3.6 times.

$\psi(3770)$  excited  $c\bar{c}$  state which decays primarily into a  $D\bar{D}$  pair.



The ability to fully reconstruct most events allows us to determine the presence of particles that don't interact in our detector ( $\nu, K_L$ ).

$D\bar{D}$  pair created in a quantum-correlated state, unique compared to generic D decays.

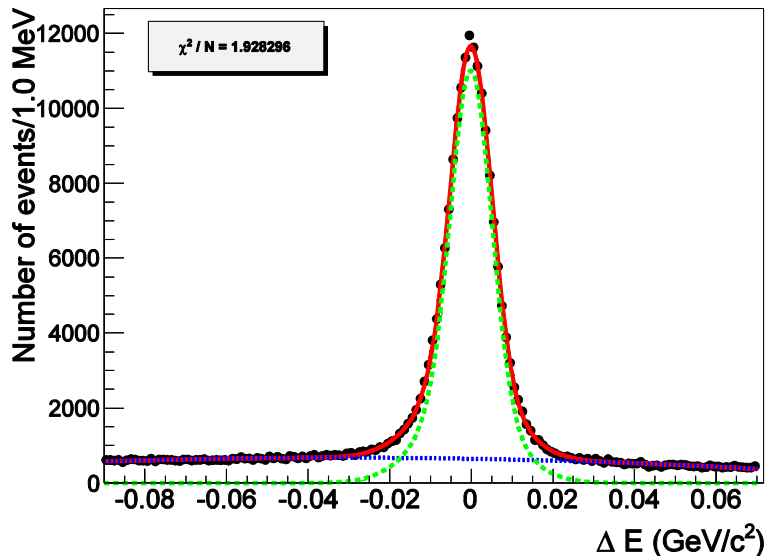
# Single D Tagging

## Single Tagging

Reconstruct particles from a single D decay.

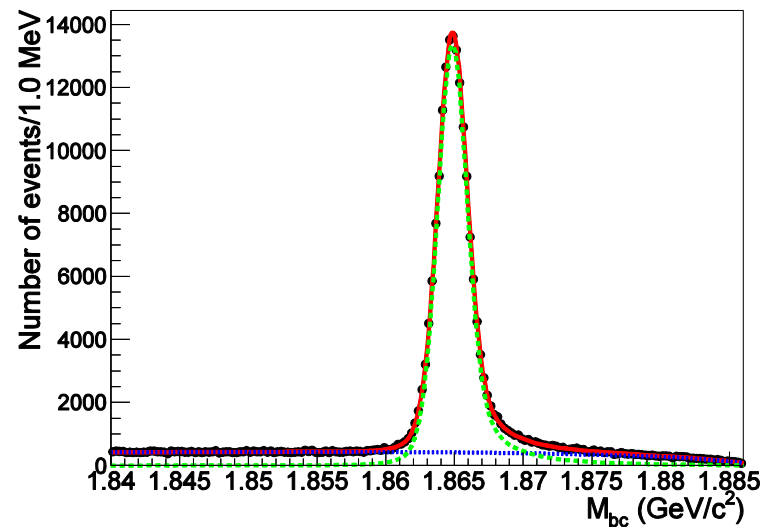
$$\Delta E = E_{D Rec} - E_{Beam}$$

$\Delta E$  Fit of  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$



$$M_{bc} = \sqrt{E_{beam}^2 - |\vec{P}_{D Rec}|^2}$$

$M_{bc}$  Fit of  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$



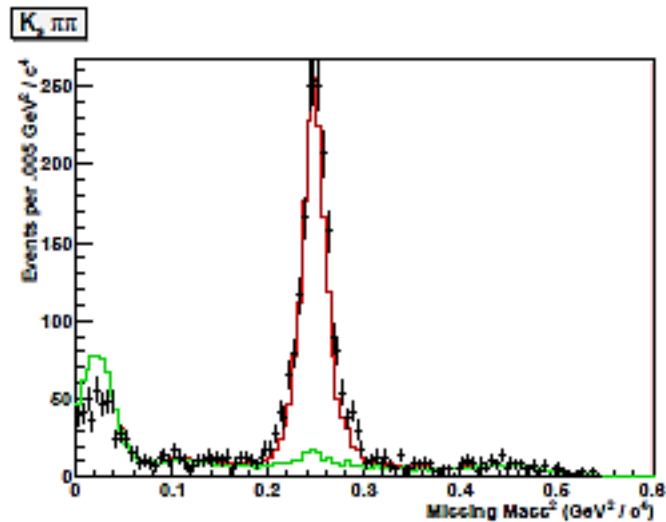
# Double D Tagging

Double Tagging reconstructs both D's

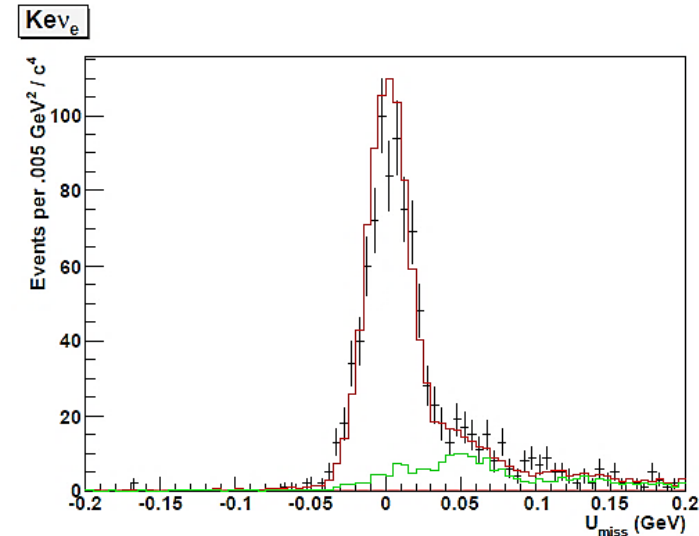
Reconstructing both D's significantly removes background ,  
however it is at the cost of statistics.

Ability to reconstruct particles that don't decay in our detector ( $K_L^0, \nu$ )

$$M_{miss}^2 \equiv E_{miss}^2 - |\vec{p}_{miss}|^2$$

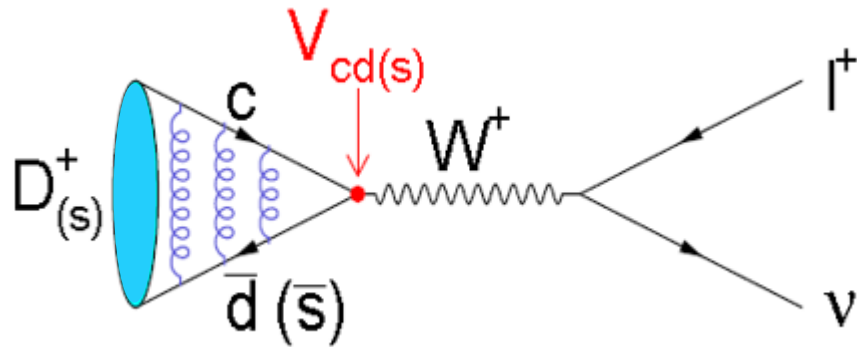


$$U_{miss} \equiv E_{miss} - |\vec{p}_{miss}|$$





# Leptonic decays



SM prediction:

$$\Gamma(D_{(s)}^+ \rightarrow l^+ \nu_l) = \frac{G_F^2 f_{D_{(s)}^+}^2}{8\pi} |V_{cd(s)}|^2 m_l^2 m_{D_{(s)}^+} \left(1 - \frac{m_l^2}{m_{D_{(s)}^+}^2}\right)^2$$

Allows us to explore precision measurements of :

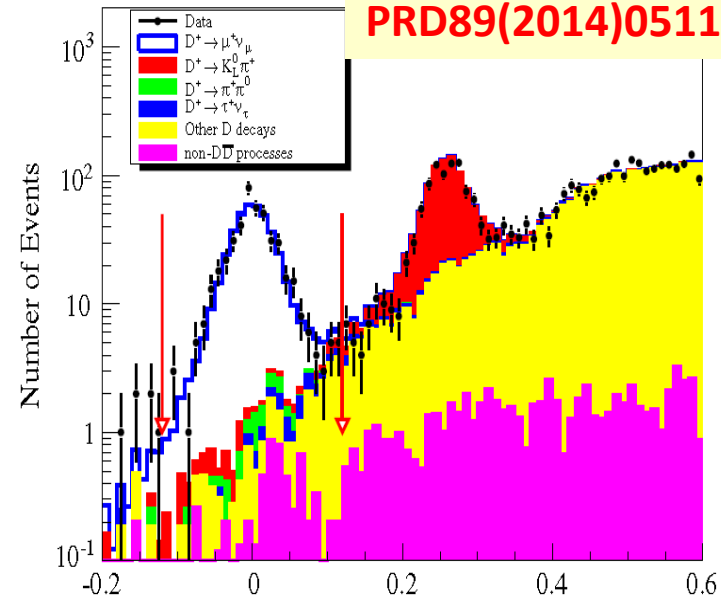
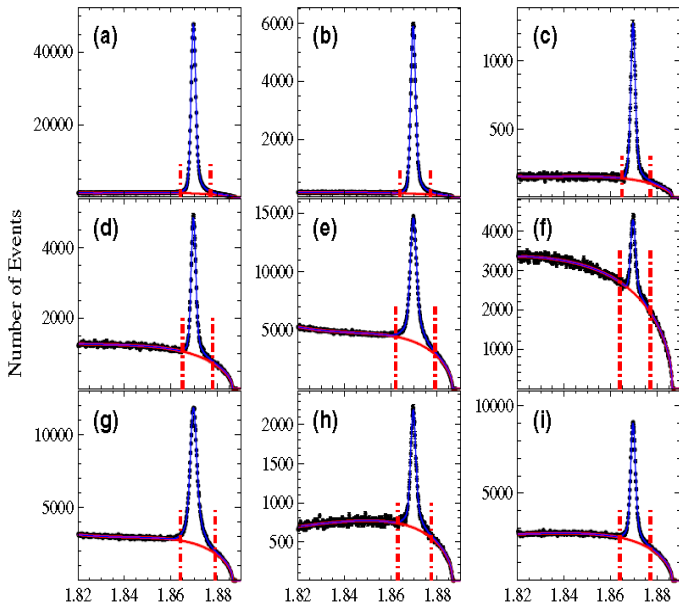
- Decay constants  $f_{D_{(s)}^+}$  using input from  $|V_{cd(s)}|^{\text{CKMfitter}}$
- CKM matrix elements  $|V_{cd(s)}|$  using input from  $f_{D_{(s)}^+}^{\text{LQCD}}$

# Measurement of $\mathcal{B}(D^+ \rightarrow \mu^+ \nu_\mu)$ , $f_{D^+}$ and $|V_{cd}|$

$e^+e^- \rightarrow \psi(3770) \rightarrow D^+D^-$

2.92 fb<sup>-1</sup> data@ 3.773 GeV

PRD89(2014)051104R



$$M_{BC} [\text{GeV}/c^2] = \sqrt{E_{beam}^2 - |\vec{P}_{D Rec}|^2}$$

$$M_{miss}^2 [\text{GeV}^2/c^4] = E_{miss}^2 - |\vec{p}_{miss}|^2$$

$$N_{D_{tag}} = (170.31 \pm 0.34) \times 10^4$$

$$\mathcal{B}[D^+ \rightarrow \mu^+ \nu] = (3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$$

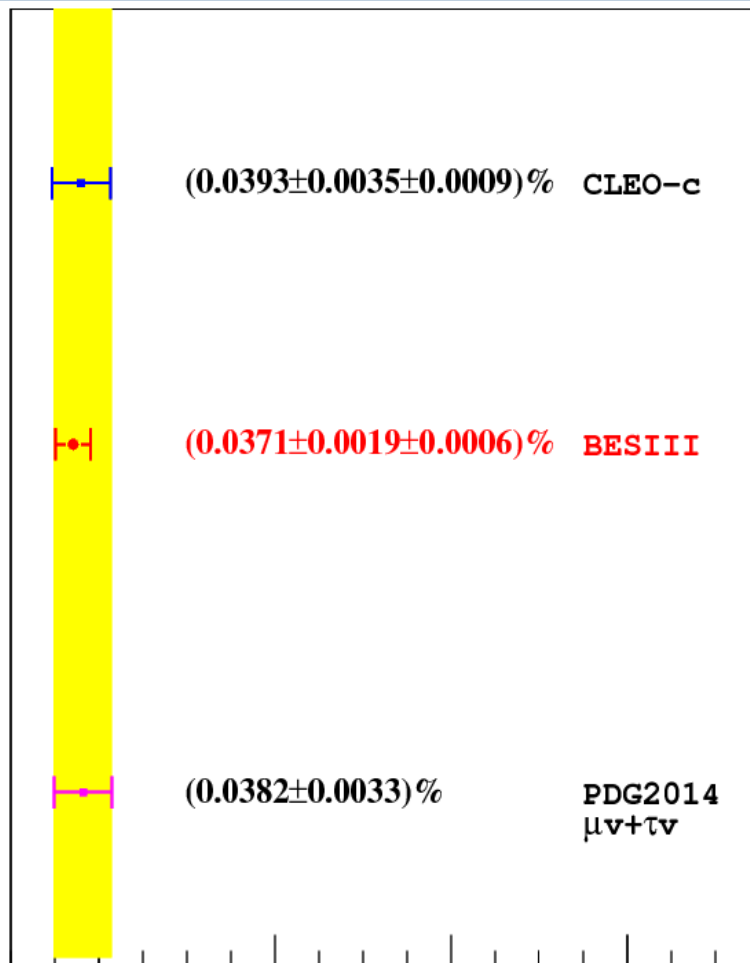
Input  $t_{D^+}$ ,  $m_{D^+}$ ,  $m_{\mu^+}$  on PDG  
and  $|V_{cd}| = 0.22520 \pm 0.00065$   
from CKM-Fitter

Input  $t_{D^+}$ ,  $m_{D^+}$ ,  $m_{\mu^+}$  on PDG and  
LQCD calculated  $f_{D^+} = 207 \pm 4$   
MeV [PRL100(2008)062002]

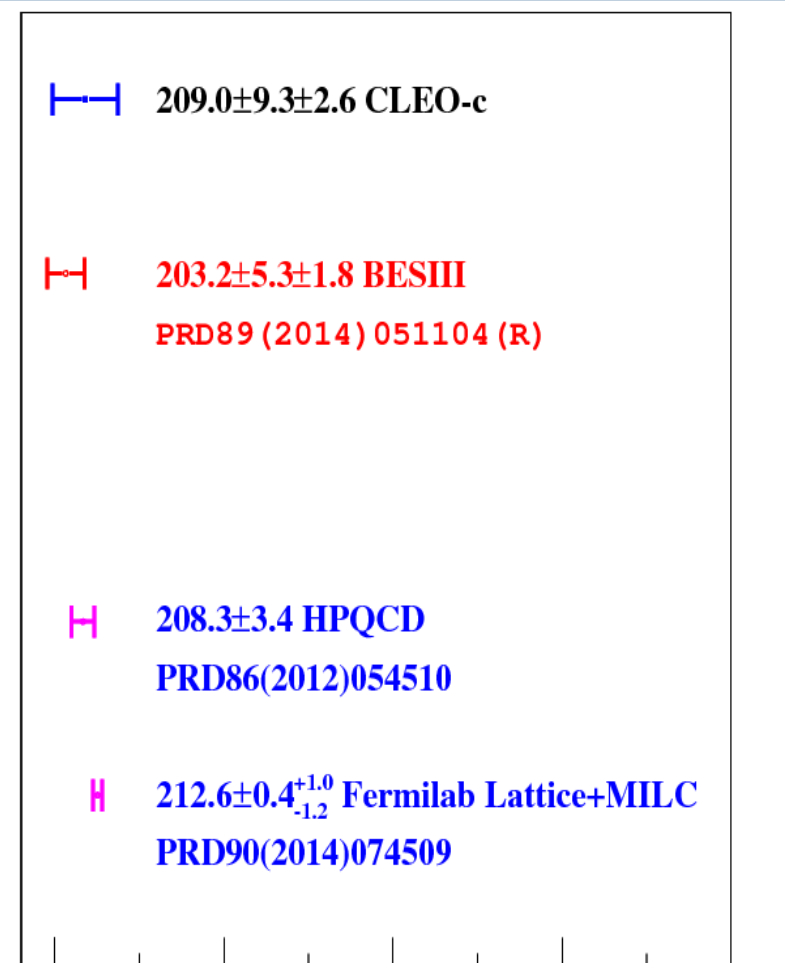
$$f_{D^+} = (203.2 \pm 5.3 \pm 1.8) \text{ MeV}$$

$$|V_{cd}| = 0.2210 \pm 0.0058 \pm 0.0047$$

# Comparison of $\mathcal{B}(D^+ \rightarrow \mu^+ \nu_\mu)$ and $f_{D^+}$

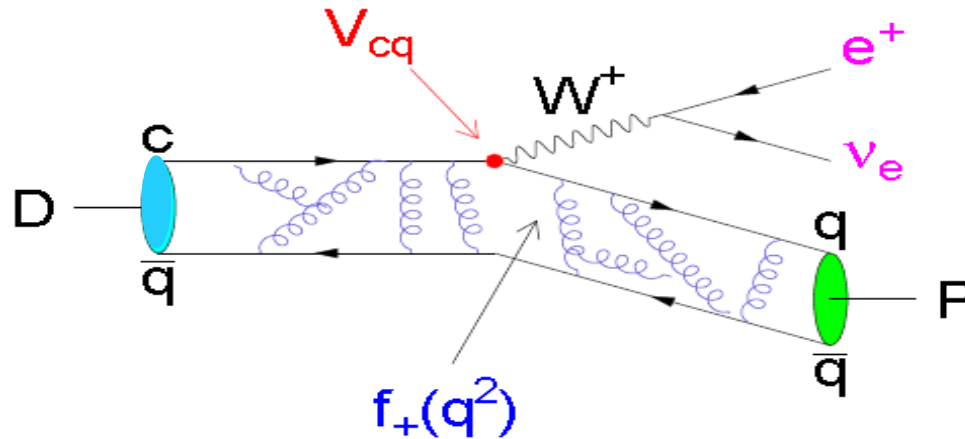


$\mathcal{B}(D^+ \rightarrow \mu^+ \nu_\mu)$



$f_{D^+}$  [MeV]

# Semi-leptonic decays: $D^0 \rightarrow K(\pi)^- e^+ \nu_e$



Differential Rates: 
$$\frac{d\Gamma(D \rightarrow K(\pi) e \nu)}{dq^2} = \frac{G_F^2 |V_{cs(d)}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

Allows us to explore precision measurements of :

- Form factors  $f_+^{D \rightarrow K(\pi)^-}(q^2)$  using input from  $|V_{cs(d)}|^{\text{CKMfitter}}$

– **Single pole form**

$$f_+(q^2) = \frac{f_+(0)}{1 - \frac{q^2}{M_{pole}^2}}$$

– **ISGW2 model**

$$f_+(q^2) = f_+(q_{max}^2) \left( 1 + \frac{r_{ISGW2}^2}{12} (q_{max}^2 - q^2) \right)^{-2}$$

– **Modified pole model**

$$f_+(q^2) = \frac{f_+(0)}{\left( 1 - \frac{q^2}{M_{pole}^2} \right) \left( 1 - \alpha \frac{q^2}{M_{pole}^2} \right)}$$

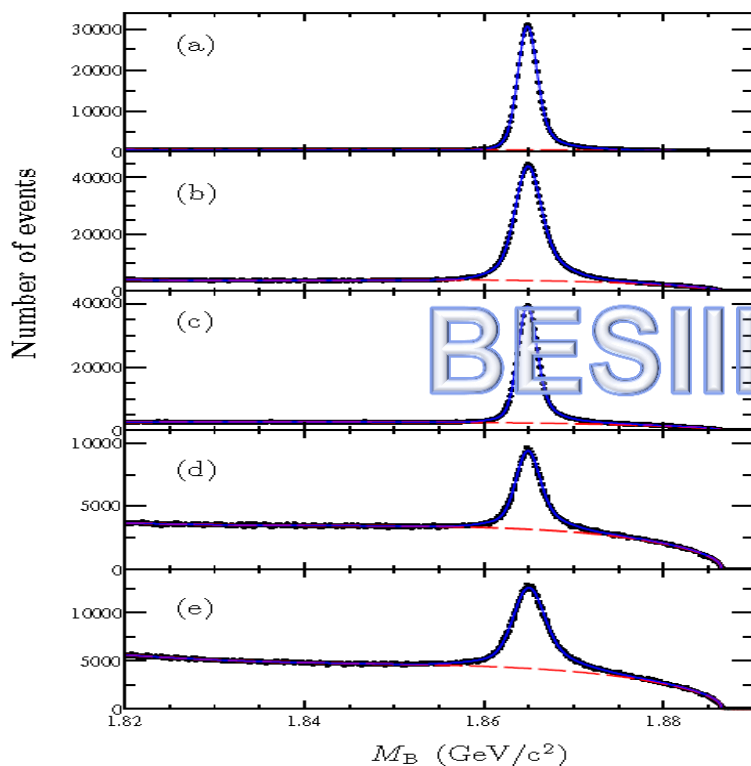
– **Series expansion model**

$$f_+(t) = \frac{1}{P(t)\Phi(t, t_0)} a_0(t_0) \left( 1 + \sum_{k=1}^{\infty} r_k(t_0) [z(t, t_0)]^k \right)$$

- CKM matrix elements  $|V_{cs(d)}|$  using input from  $f^{LQCD}_{D^0}$

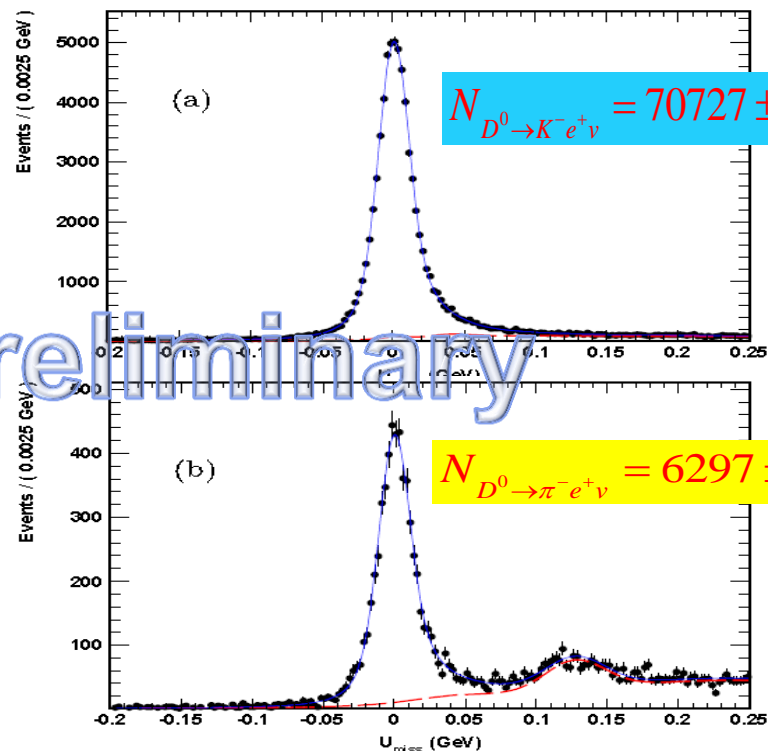
# Measurement of $\mathcal{B}(D^0 \rightarrow K(\pi)^- e^+ \nu_e)$

$e^+e^- \rightarrow \psi(3770) \rightarrow D^0 D^0$



$$N_{D_{\text{tag}}^0} = (279.33 \pm 0.37) \times 10^4$$

$U_{\text{miss}} = E_{\text{miss}} - P_{\text{miss}}$



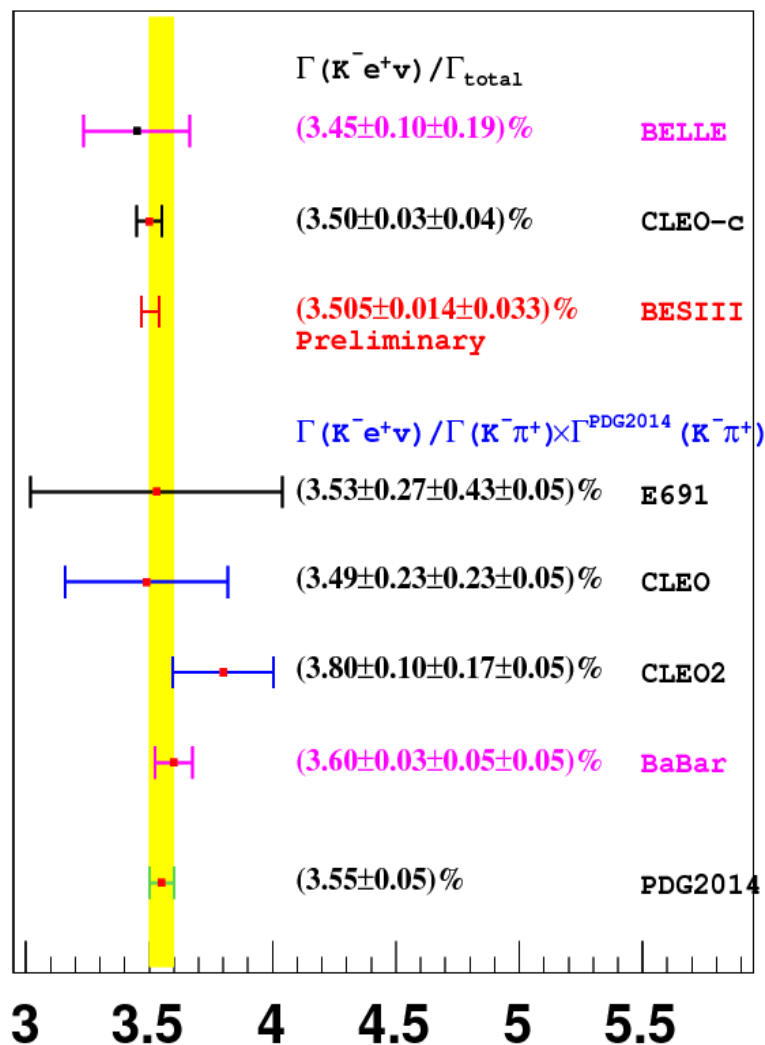
$$N_{D^0 \rightarrow K^- e^+ \nu} = 70727 \pm 278$$

$$N_{D^0 \rightarrow \pi^- e^+ \nu} = 6297 \pm 87$$

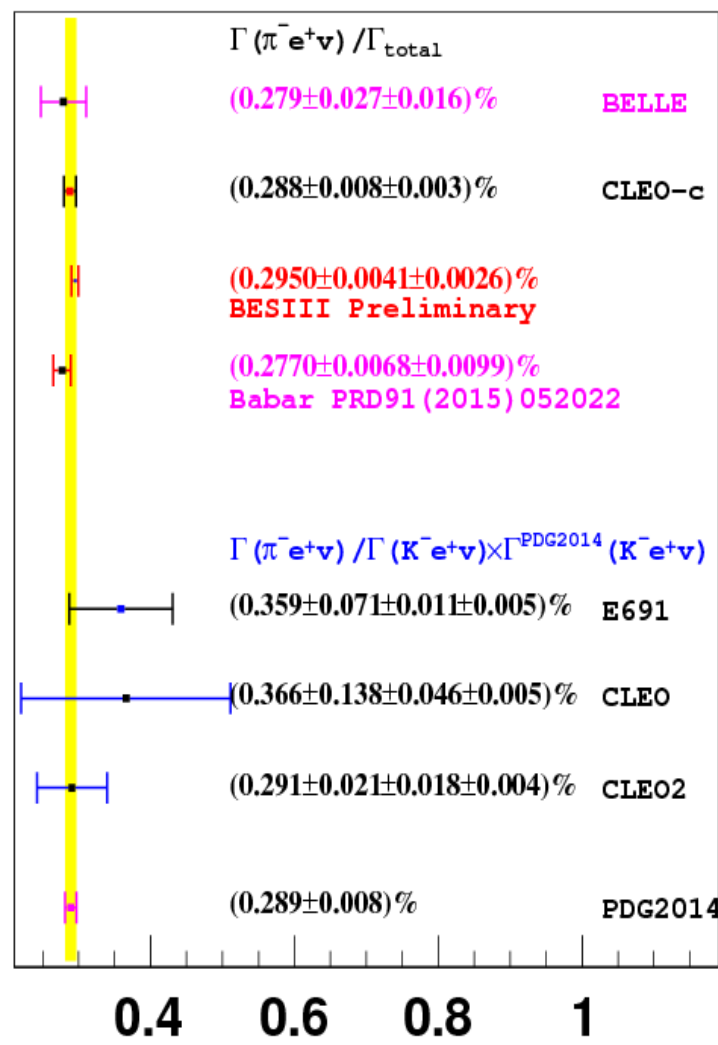
$$\mathcal{B}_{D^0 \rightarrow K^- e^+ \nu} = (3.505 \pm 0.014 \pm 0.033)\%$$

$$\mathcal{B}_{D^0 \rightarrow \pi^- e^+ \nu} = (0.2950 \pm 0.0041 \pm 0.0026)\%$$

# Comparison of $\mathcal{B}(D^0 \rightarrow K(\pi)^- e^+ \nu_e)$

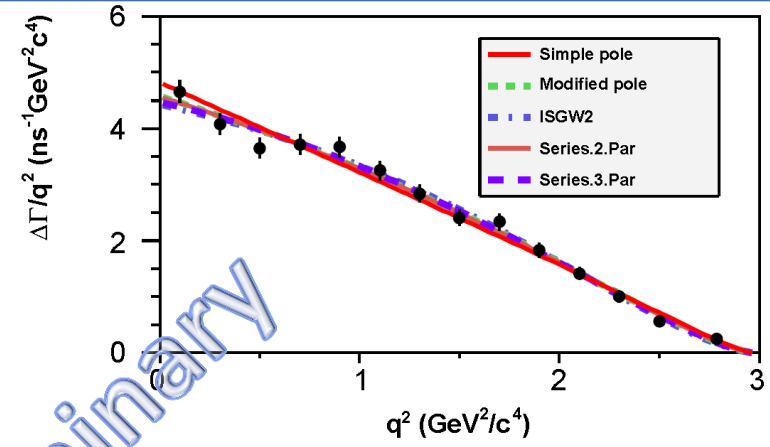
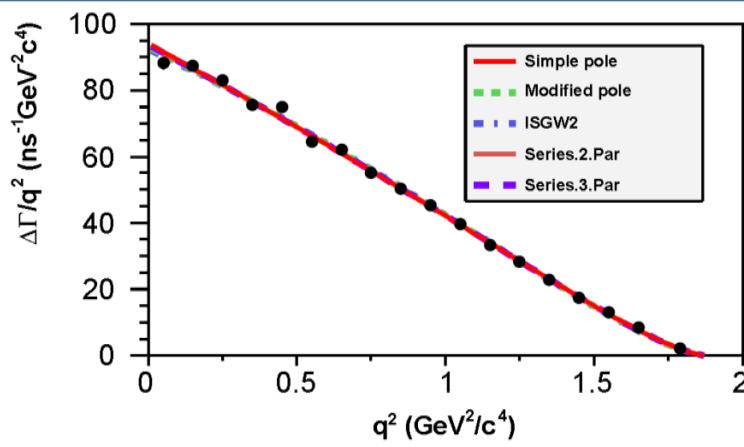


$\mathcal{B}(D^0 \rightarrow K^- e^+ \nu_e)$



$\mathcal{B}(D^0 \rightarrow \pi^- e^+ \nu_e)$

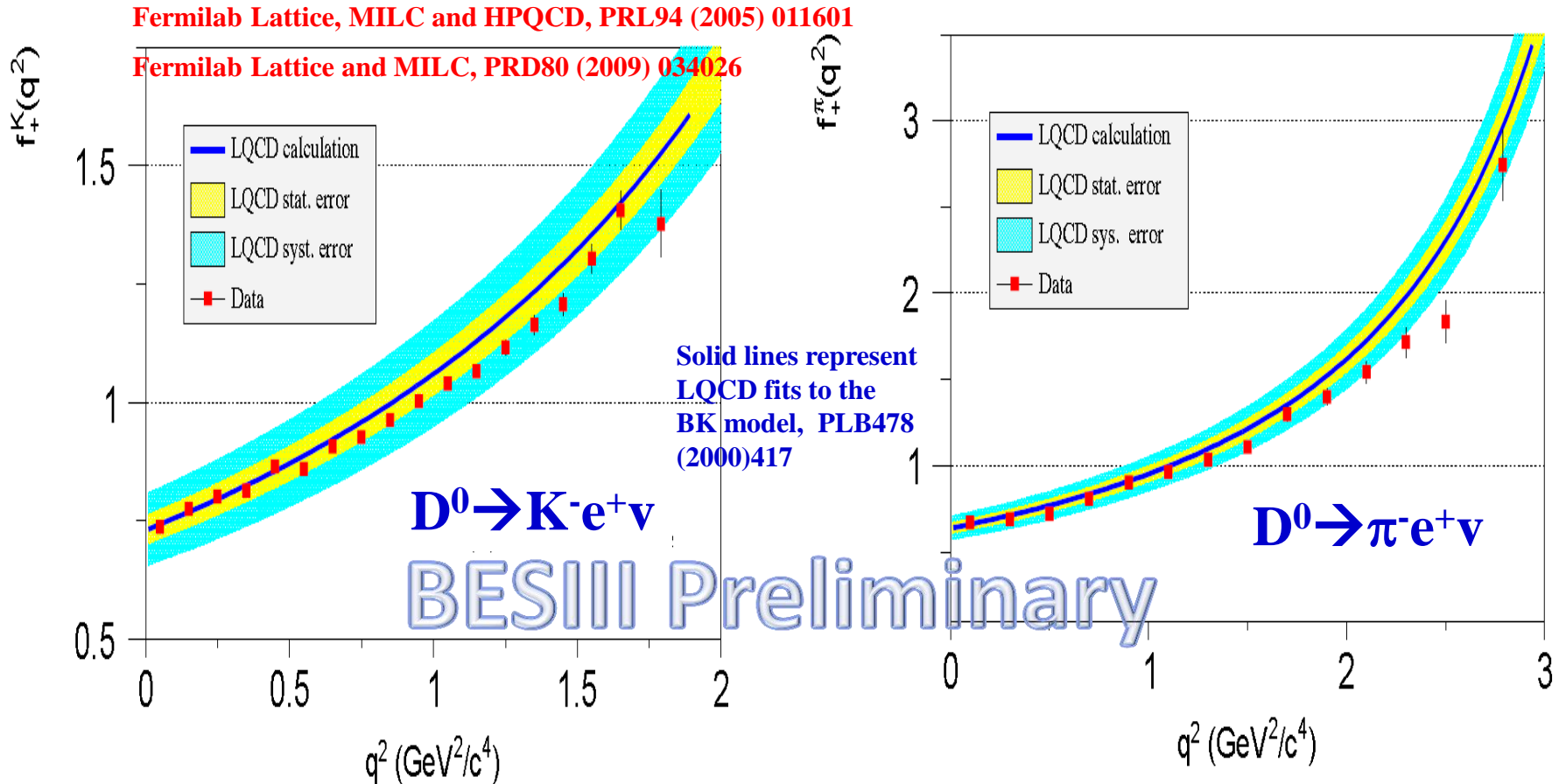
# Extracted Parameters of Form Factors



		$D^0 \rightarrow K^+ e^- \nu$		$D^0 \rightarrow \pi^+ e^- \nu$
<b>Simple Pole</b>	$f_K^+(0) V_{cs} $	$0.7209 \pm 0.0022 \pm 0.0033$	$f_\pi^+(0) V_{cd} $	$0.1475 \pm 0.0014 \pm 0.0005$
	$M_{\text{pole}}$	$1.9207 \pm 0.0103 \pm 0.0069$	$M_{\text{pole}}$	$1.9114 \pm 0.0118 \pm 0.0038$
<b>Mod. Pole</b>	$f_K^+(0) V_{cs} $	$0.7163 \pm 0.0024 \pm 0.0034$	$f_\pi^+(0) V_{cd} $	$0.1437 \pm 0.0017 \pm 0.0008$
	$\alpha$	$0.3088 \pm 0.0195 \pm 0.0129$	$\alpha$	$0.2794 \pm 0.0345 \pm 0.0113$
<b>ISGW2</b>	$f_K^+(0) V_{cs} $	$0.7139 \pm 0.0023 \pm 0.0034$	$f_\pi^+(0) V_{cd} $	$0.1415 \pm 0.0016 \pm 0.0006$
	$r_{\text{ISGW2}}$	$1.6000 \pm 0.0141 \pm 0.0091$	$r_{\text{ISGW2}}$	$2.0688 \pm 0.0394 \pm 0.0124$
<b>Series.2.Par</b>	$f_K^+(0) V_{cs} $	$0.7172 \pm 0.0025 \pm 0.0035$	$f_\pi^+(0) V_{cd} $	$0.1435 \pm 0.0018 \pm 0.0009$
	$r_1$	$-2.2278 \pm 0.0864 \pm 0.0575$	$r_1$	$-2.0365 \pm 0.0807 \pm 0.0260$
<b>Series.3.Par</b>	$f_K^+(0) V_{cs} $	$0.7196 \pm 0.0035 \pm 0.0041$	$f_\pi^+(0) V_{cd} $	$0.1420 \pm 0.0024 \pm 0.0010$
	$r_1$	$-2.3331 \pm 0.1587 \pm 0.0804$	$r_1$	$-1.8434 \pm 0.2212 \pm 0.0690$
	$r_2$	$3.4223 \pm 3.9090 \pm 2.4092$	$r_2$	$-1.3871 \pm 1.4615 \pm 0.4677$

# Measurement of $f_+^{K(\pi)}(q^2)$

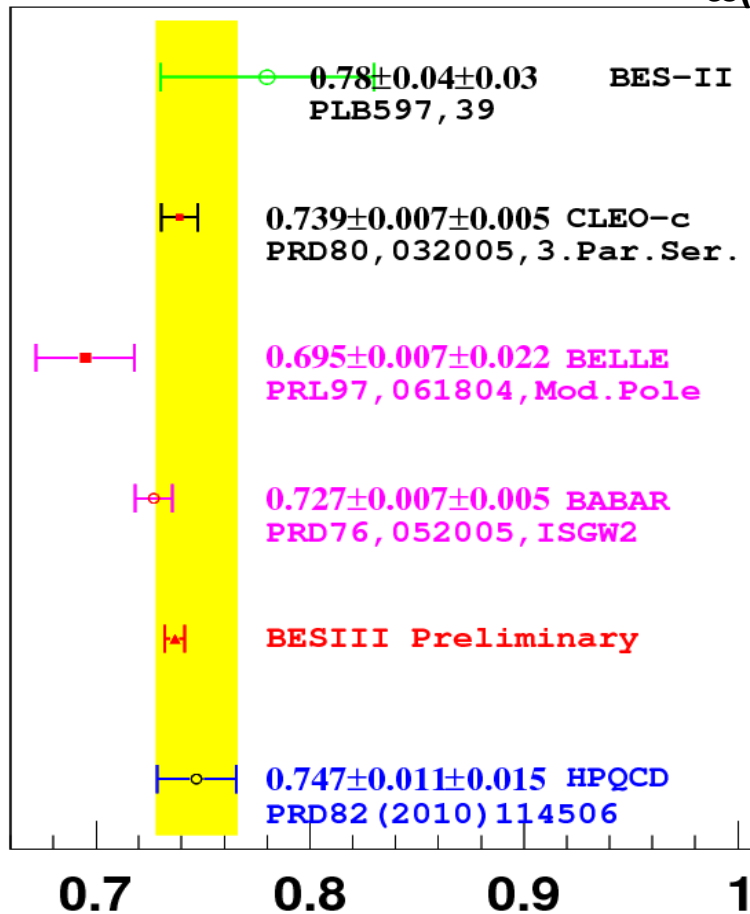
## Experimental data calibrate LQCD calculation



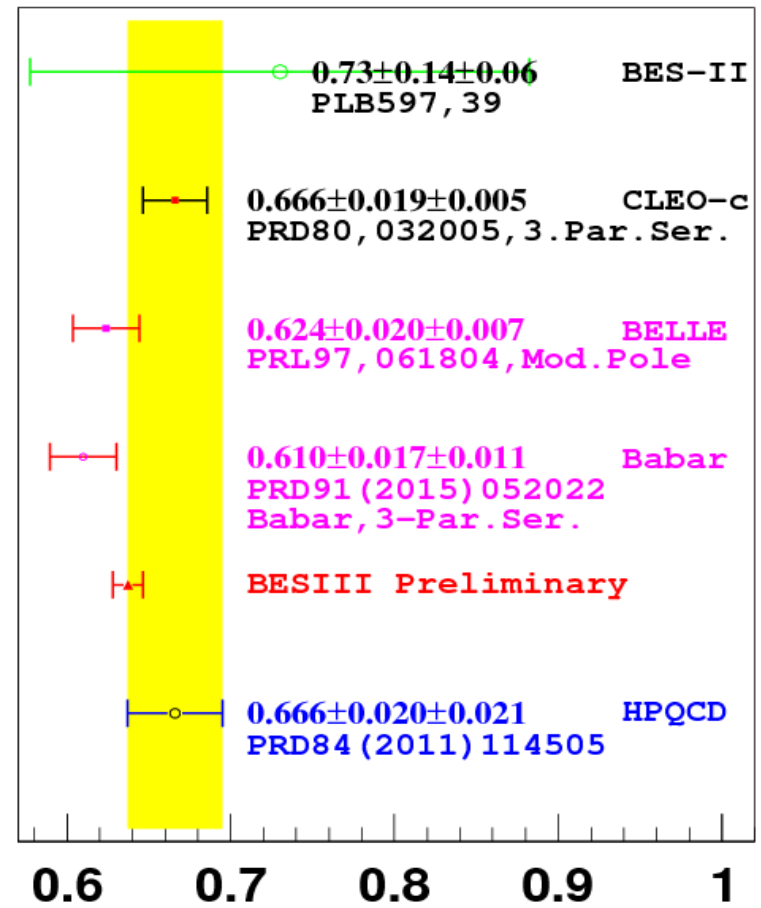


# Improved Form Factor at $q=0$

## Input $|V_{cs(d)}|$ from CKM-Fitter



$f_+^K(0)$



$f_+^\pi(0)$

# Improved $|V_{cs(d)}|$ at BESIII

■ Method 1

$$B[D_{(s)}^+ \rightarrow l^+ \nu]$$

Input  $t_{D^+}$ ,  $m_{D^+}$ ,  $m_{\mu^+}$  on PDG and  
LQCD calculated  $f_{D(s)^+}$

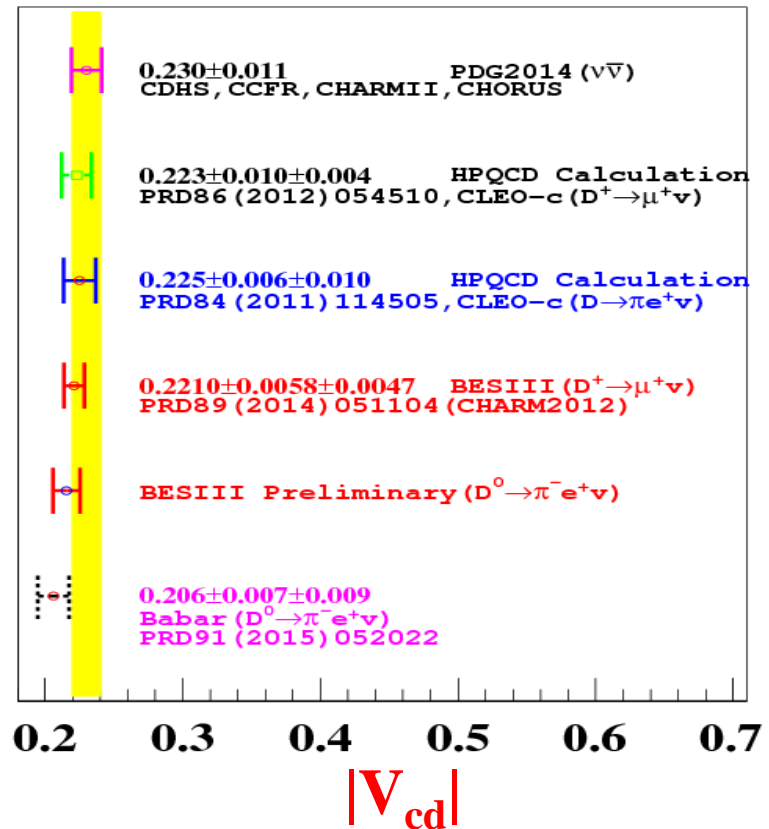
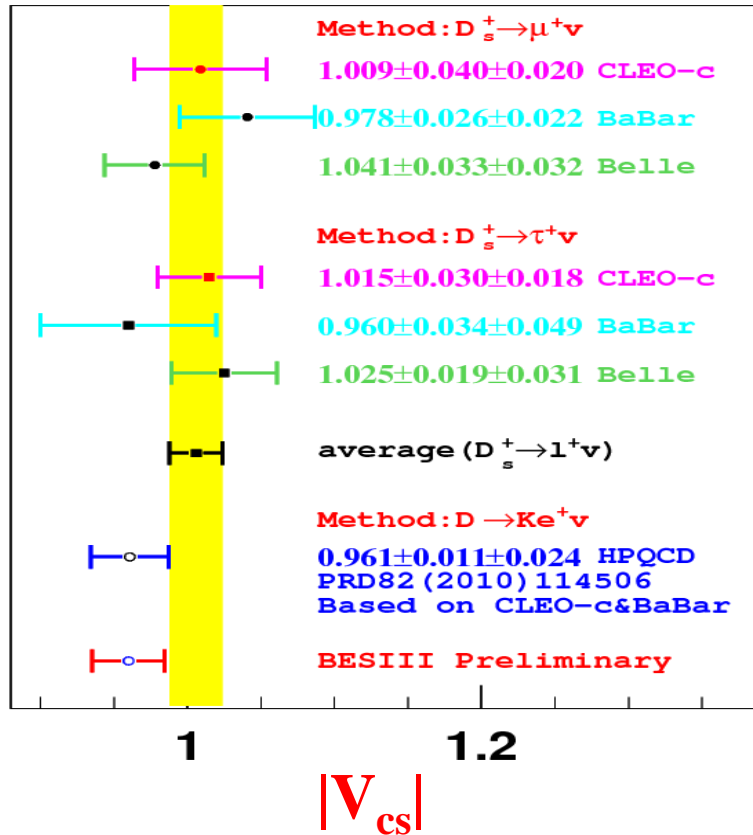
$$|V_{cd(s)}|$$

■ Method 2

$$f^{D \rightarrow K(\pi)^+}_+(0) |V_{cs(d)}|$$

Input  $f^{D \rightarrow K(\pi)^+}_+(0)$  of LQCD

$$|V_{cs(d)}|$$



# Definition of $Y_{CP}$ in $D^0\bar{D}^0$ oscillation

Oscillations in  $D^0\bar{D}^0$  system are characterized by two mixing parameters

$$x = \Delta m / \Gamma$$

$$y = \Delta \Gamma / 2\Gamma$$

Where  $\Delta m$  and  $\Delta \Gamma$  are the mass and width differences between the two mass eigenstates

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$$

$$\phi = \arg(p/q)$$

The D meson's CP eigenstates can be written as

$$|D_{CP\pm}\rangle = \frac{|D^0\rangle \pm |\bar{D}^0\rangle}{\sqrt{2}}$$

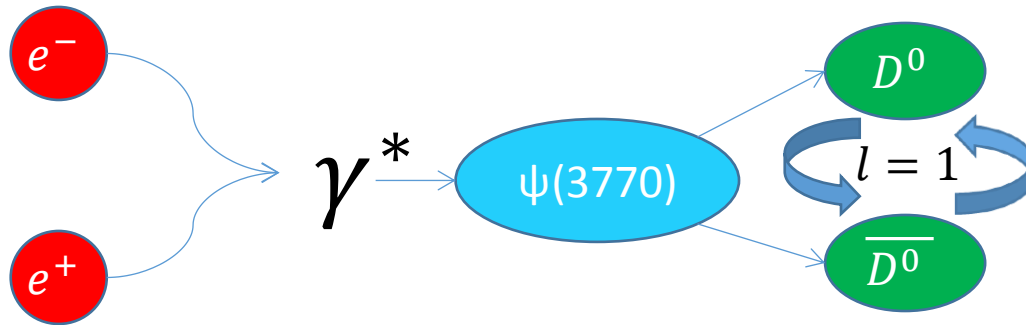
Allowing for small indirect CPV, the  $y$  parameter of the CP eigenstates becomes

$$y_{cp} = \frac{1}{2} \left[ y \cos \phi \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) - x \sin \phi \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \right]$$

In absence of CPV :

- $y_{cp} = y$
- $\left| \frac{q}{p} \right| = 1$
- $\phi = 0$

# Using Semi-leptonic decays to measure $Y_{CP}$



If  $D^0$  in CP eigenstate,  
 $\bar{D}^0$  must be in opposite  
 CP eigenstate

Total decay width of CP eigenstates:

$$\Gamma_{CP\pm} = \Gamma(1 \mp y_{CP})$$

Semi-leptonic ( $D \rightarrow l$ ) decay width is only sensitive to flavor content

Therefore, Semi-leptonic decay from a CP eigenstate

$$\mathcal{B}_{D_{CP\pm} \rightarrow l} \approx \mathcal{B}_{D \rightarrow l}(1 \mp y_{CP})$$

$$\therefore y_{CP} \approx \frac{1}{4} \left( \frac{\mathcal{B}_{D_{CP-} \rightarrow l}}{\mathcal{B}_{D_{CP+} \rightarrow l}} - \frac{\mathcal{B}_{D_{CP+} \rightarrow l}}{\mathcal{B}_{D_{CP-} \rightarrow l}} \right)$$

This can be obtained through Single and double tag yields and efficiencies

$$\mathcal{B}_{D_{CP\mp} \rightarrow l} = \frac{N_{CP\pm;l}}{N_{CP\pm}} \cdot \frac{\epsilon_{CP\pm}}{\epsilon_{CP\pm;l}}$$

# $Y_{CP}$ Results

$$\mathcal{B}_{D_{CP\mp} \rightarrow l} = \frac{N_{CP\pm;l}}{N_{CP\pm}} \cdot \frac{\epsilon_{CP\pm}}{\epsilon_{CP\pm;l}}$$

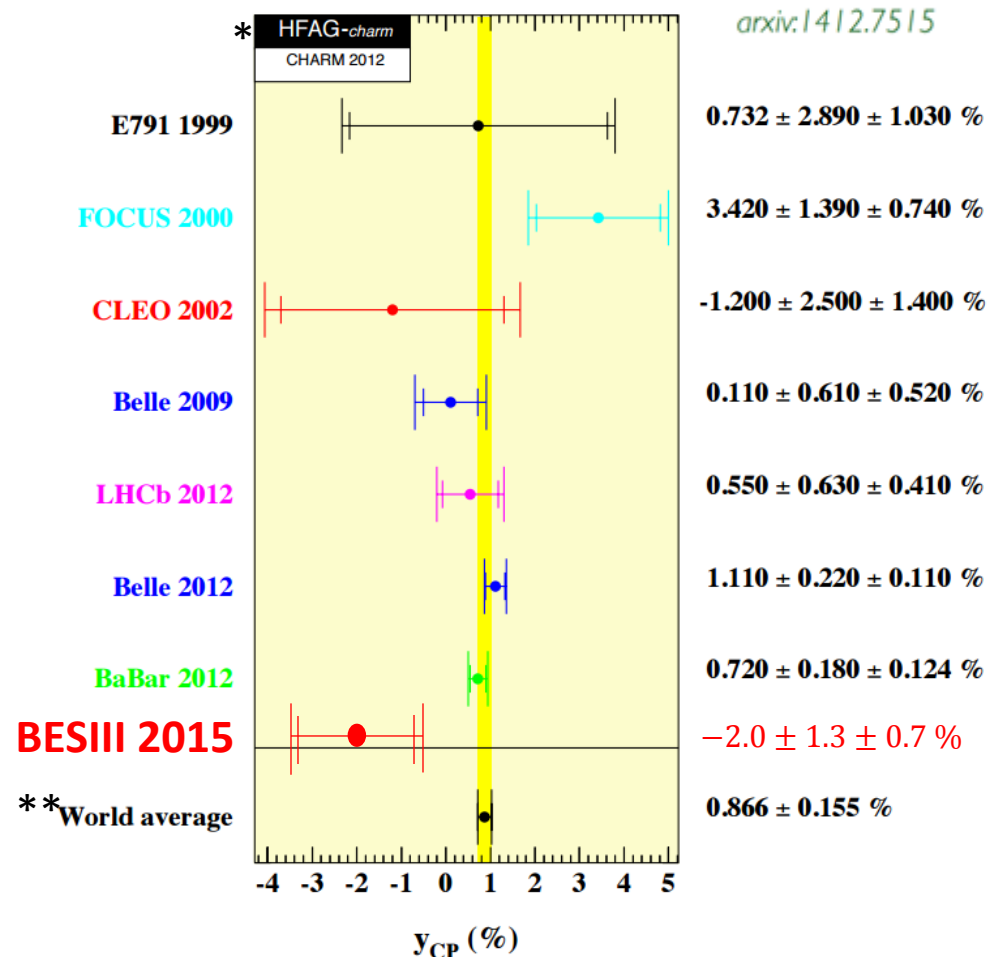
Branching ratios of  $K_{e\nu}$  and  $K_{\mu\nu}$  are combined to get  $\mathcal{B}_{D_{CP\mp} \rightarrow l}$

Results are combined from different CP modes into a global fit using standard weighted least-square method.

Result:

$$y_{cp} = (-2.0 \pm 1.3_{(stat)} \pm 0.7_{(sys)})\%$$

Phys.Lett.B 744(2015) 339-346

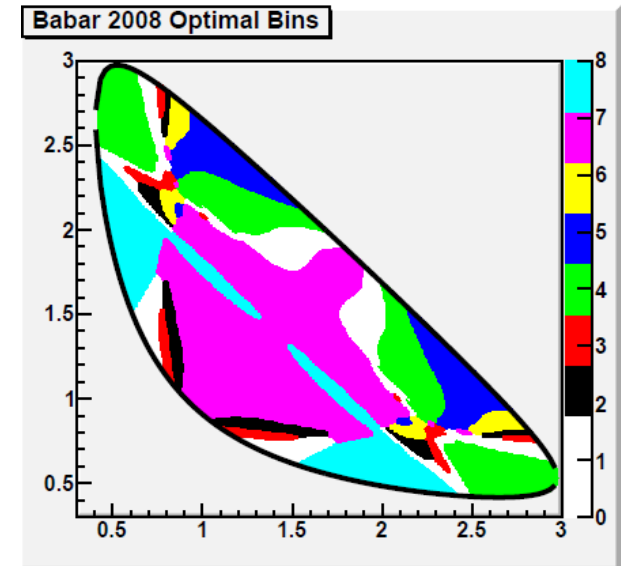


\* Edited to compare with BESIII result

\*\* BESIII not included in world average

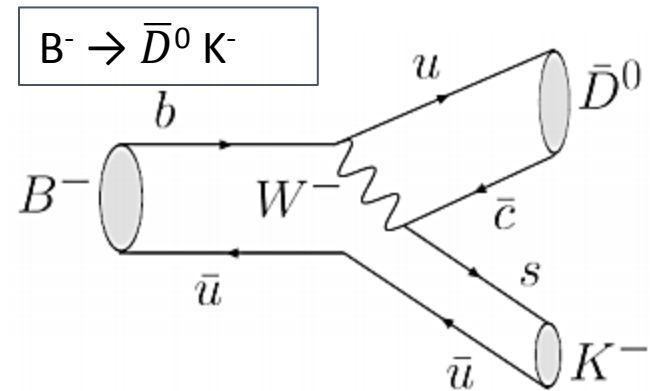
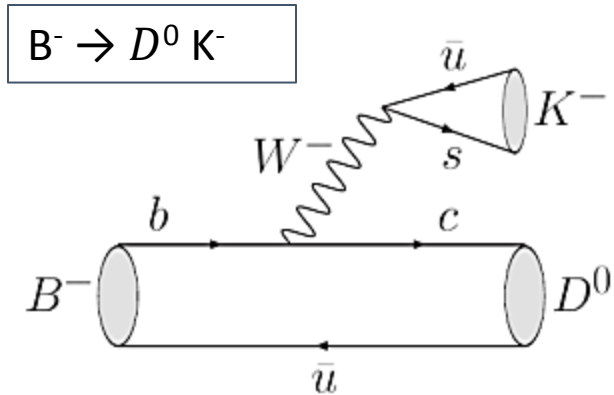
# $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ Strong-Phase Parameters

Measuring the Dalitz binned  
strong-phase difference between  
 $D^0$  and  $\overline{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$



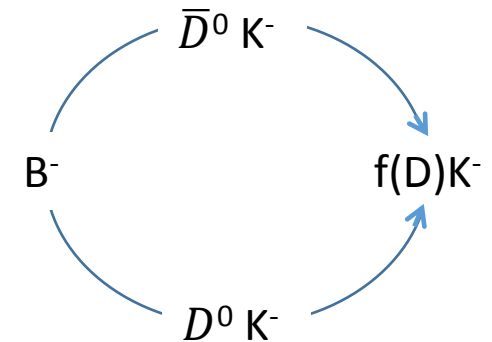
This measurement is important for reducing the systematic/model uncertainty of the measurement of the CKM UT angle  $\gamma$  done at the B factories using the GGSZ method.

# CKM UT angle $\gamma$ measurement



$$\frac{\langle B^- \rightarrow \bar{D}^0 K^- \rangle}{\langle B^- \rightarrow D^0 K^- \rangle} = r_B e^{i(\delta_B - \gamma)}$$

Determine  $\gamma$  through the measurement of the interference between  $b \rightarrow c$  and  $b \rightarrow u$  transitions when  $D^0$  and  $\bar{D}^0$  both decay to the same final state  $f(D)$ .



# GGSZ method of measuring $\gamma$

Binned decay rate:

$$\begin{aligned}\Gamma(B^\pm \rightarrow D(K_S \pi^+ \pi^-)K^\pm)_i &= T_i + r_B^2 T_{-i} + 2r_B \sqrt{T_i T_{-i}} \cos(\delta_B \pm \gamma - \Delta\delta_D) \\ &= T_i + r_B^2 T_{-i} + 2r_B \sqrt{T_i T_{-i}} \{c_i \cos(\delta_B \pm \gamma) + s_i \sin(\delta_B \pm \gamma)\}\end{aligned}$$

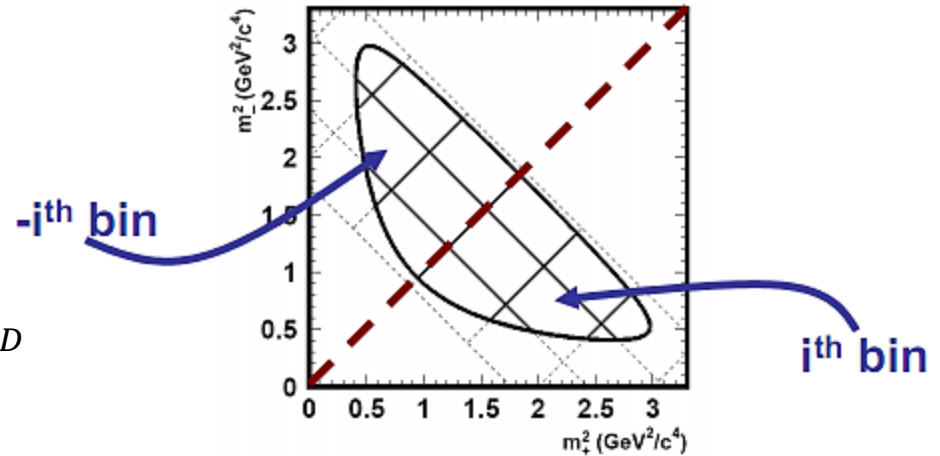
Distribution sensitive to variables:

$T_i$  : Bin yield measured in flavor decays

$r_B$  : color suppression factor  $\sim 0.1$

$\delta_B$  : strong phase of B decay

$c_i, s_i$  : weighted average of  $\cos(\Delta\delta_D)$  and  $\sin(\Delta\delta_D)$  respectively where  $\Delta\delta_D$  is the difference between phase of  $D^0$  and  $\bar{D}^0$



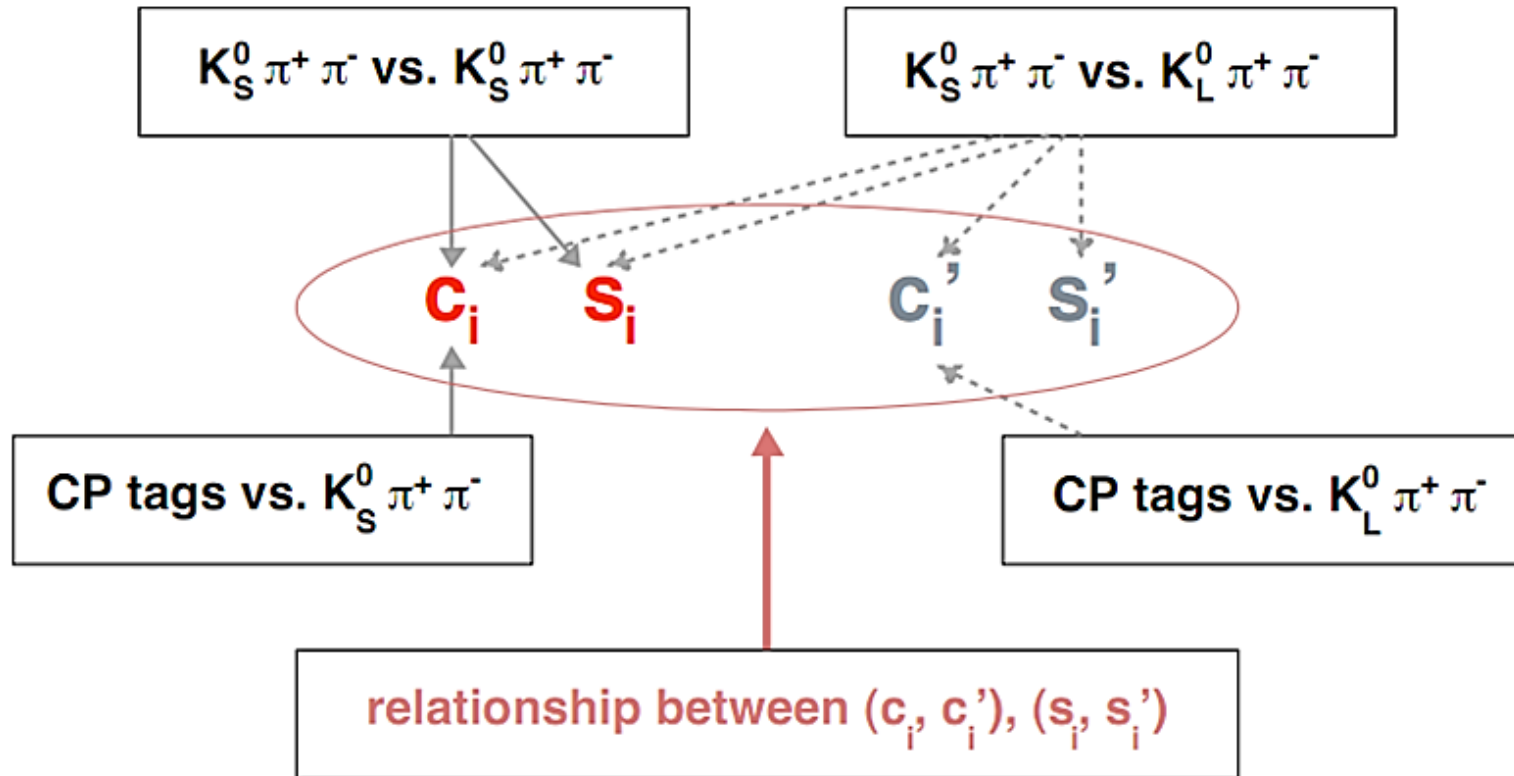
Mirrored binning over  $x=y$  makes it so  $c_i = c_{-i}$  and  $s_i = -s_{-i}$

$T_i, r_B, \delta_B$  are measured at B-Factories

$c_i$  and  $s_i$  can be found through  $K_S \pi^+ \pi^-$  Analysis



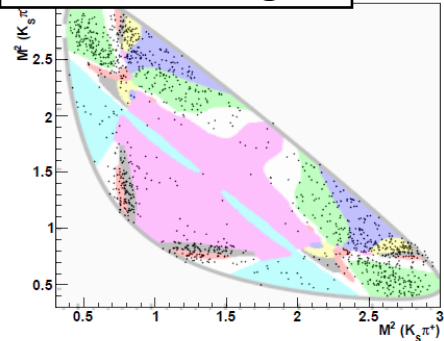
# Calculation of strong phase parameters



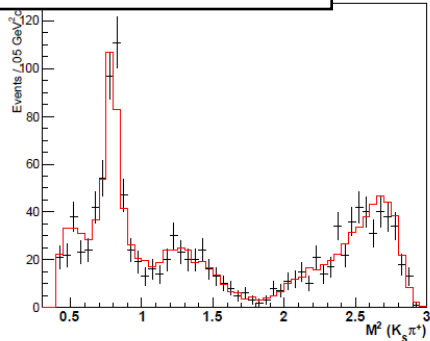
It can be shown that the strong phase parameters can be determined through yields and efficiencies of ST and DT modes.

# $K_S^0 \pi^+ \pi^-$ vs CP tags

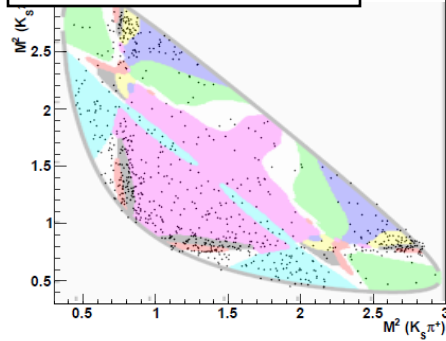
CP Even Tags



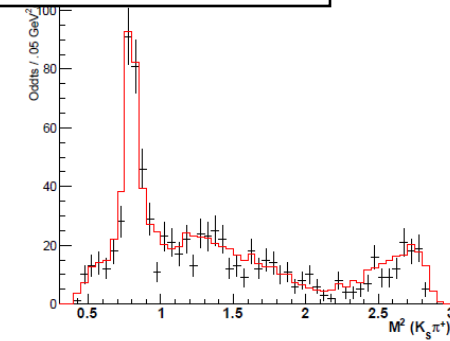
Y Projection:  $K_S^0 \pi^-$



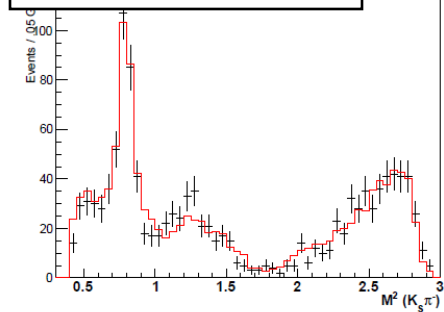
CP Odd Tags



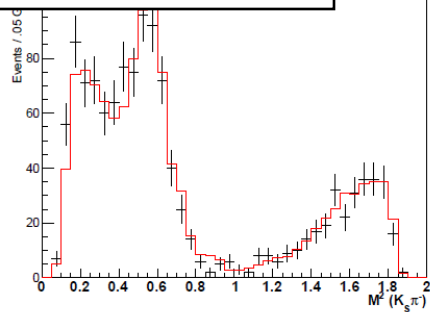
Y Projection:  $K_S^0 \pi^-$



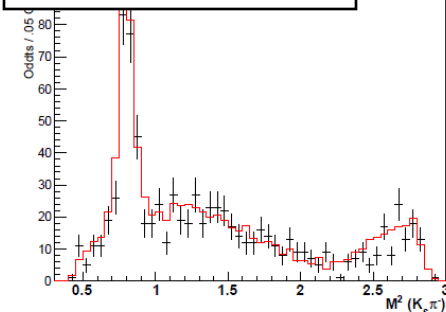
X Projection:  $K_S^0 \pi^+$



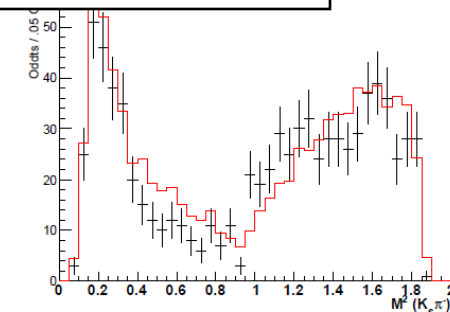
$\pi^+ \pi^-$  Mass<sup>2</sup>



X Projection:  $K_S^0 \pi^+$



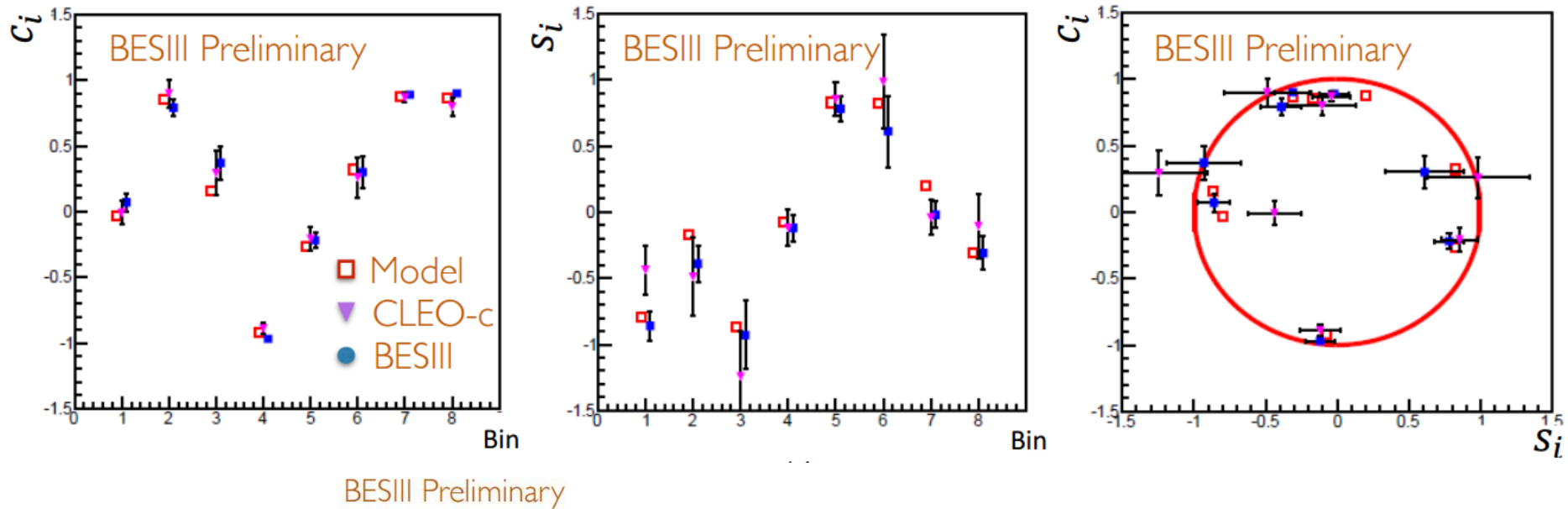
$\pi^+ \pi^-$  Mass<sup>2</sup>



BESIII  
Preliminary

Type	Tag List
Pseudo-Flavored	$K^- \pi^+, K^- \pi^+ \pi^0, K^- \pi^+ \pi^+ \pi^-$
$S^+$	$K^+ K^-, \pi^+ \pi^-, K_S \pi^0 \pi^0, K_L \pi^0$
$S^-$	$K_S \pi^0, K_S \eta (\rightarrow \gamma \gamma), K_S \eta (\rightarrow \pi^+ \pi^- \pi^0), K_S \omega, K_S \eta'$

# Strong-phase parameters results



Bins	$c_i$		$s_i$	
	BES-III	CLEO-c	BES-III	CLEO-c
1	$0.066 \pm 0.066$	$-0.009 \pm 0.088$	$-0.843 \pm 0.119$	$-0.438 \pm 0.184$
2	$0.796 \pm 0.061$	$0.900 \pm 0.106$	$-0.357 \pm 0.148$	$-0.490 \pm 0.295$
3	$0.361 \pm 0.125$	$0.292 \pm 0.168$	$-0.962 \pm 0.258$	$-1.243 \pm 0.341$
4	$-0.985 \pm 0.017$	$-0.890 \pm 0.041$	$-0.090 \pm 0.093$	$-0.119 \pm 0.141$
5	$-0.278 \pm 0.056$	$-0.208 \pm 0.085$	$0.778 \pm 0.092$	$0.853 \pm 0.123$
6	$0.267 \pm 0.119$	$0.258 \pm 0.155$	$0.635 \pm 0.293$	$0.984 \pm 0.357$
7	$0.902 \pm 0.017$	$0.869 \pm 0.034$	$-0.018 \pm 0.103$	$-0.041 \pm 0.132$
8	$0.888 \pm 0.036$	$0.798 \pm 0.070$	$-0.301 \pm 0.140$	$-0.107 \pm 0.240$

CLEO-c result: *Phys. Rev. D* 82, 112006

- Reduction in the  $c_i s_i$  contribution to the uncertainty in  $\gamma$  of  $\sim 40\%$
- Improved statistics from B factories could place uncertainty from the  $c_i s_i$  contribution at  $\sim 1\%$

# More Upcoming Analysis

Leptonic:

- $D_s^+ \rightarrow l^+ \nu$  analysis

Semi-leptonic:

- $D^+ \rightarrow K_L e^+ \nu$
- $D^+ \rightarrow (\omega, \phi) e^+ \nu$
- $D^+ \rightarrow K^- \pi^+ e^+ \nu$

Hadronic:

- Dalitz analysis  $K_S K K$
- $D_S^+ \rightarrow \eta' X$  and  $D_S^+ \rightarrow \eta' \rho^+$
- $\sigma(e^+ e^+ \rightarrow D \bar{D})$  at  $E_{\text{cm}} = 3.773$  GeV
- $\sigma(e^+ e^+ \rightarrow D \bar{D})$  Line shape near  $E_{\text{cm}} = 3.773$  GeV

# Summary

Through the leptonic decay of  $D^+ \rightarrow \mu^+ \nu_\mu$  and the semi-leptonic decay  $D^0 \rightarrow K(\pi)^- e^+ \nu_e$ , we have obtained improved measurements of decay constant  $f_{D^+}$  and form factors  $f_+^{D \rightarrow K(\pi)}(q^2)$ , which are important to test and calibrate LQCD calculations accurately.

Leptonic and semi-leptonic decays improved measurements of CKM matrix elements, which are important for unitarity test of the CKM matrix.

Measurement of  $y_{CP}$ , the mixing parameter in  $D^0 \bar{D}^0$  oscillation, is a competitive measurement which takes advantage of the quantum correlation of the  $\psi(3770)$  dataset.

Measurement of parameters of the strong-phase difference between  $D^0$  and  $\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$ , which are important for reducing systematics in CKM UT angle  $\gamma$  measurements.

Many more BESIII analysis are on their way and we look forward to sharing our results.

# $K_S^0 \pi^+ \pi^-$ Calculation of $c_i, s_i$

For CP tag vs  $K_S^0 \pi^+ \pi^-$ , we are able to find  $c_i$

$$M_i^\pm = \frac{S_\pm}{2S_f} (K_i \pm 2c_i \sqrt{K_i K_{-i}} + K_{-i})$$

$M_i^+ (M_i^-)$  yields in each bin of Dalitz plot for CP even(odd) modes.

$S_+ (S_-)$  number of single tags for CP even(odd) modes.

$S_f$  number of single tags for flavor modes.

$K_i (K_{-i})$ , yields in each bin of Dalitz plot in flavor modes.

From the Double Dalitz modes, we are able to find  $c_i, s_i$

$$M_{i,j} = \frac{N_{D,\bar{D}}}{2S_f^2} \left( K_i K_{-j} + K_{-i} K_j - 2 \sqrt{K_i K_{-j} K_{-i} K_j} (c_i c_j + s_i s_j) \right)$$

$M_{i,j}$  yields in bin  $i$  of first Dalitz plot and bin  $j$  of second Dalitz plot.

$S_f$  number of single tags for flavor modes.

$N_{D,\bar{D}}$  total number of  $D^0 \bar{D}^0$  events.

$K_i (K_{-i})$ , yields in each bin of Dalitz plot in flavor modes.

# $K_L^0 \pi^+ \pi^-$ Calculation of $c_i, c'_i, s_i, s'_i$

For CP tag vs  $K_L^0 \pi^+ \pi^-$ , we are able to find  $c'_i$

' indicates numbers from  $K_L \pi^+ \pi^-$  decays

$$M'_i{}^\pm = \frac{S_\pm}{2S_f} \left( K'_i \mp 2c'_i \sqrt{K'_i K'_{-i} + K'_{-i}} \right)$$

$M'_i{}^+(M'_i{}^-)$  yields in each bin of Dalitz plot for CP even(odd) modes.  
 $S_+(S_-)$  number of single tags for CP even(odd) modes.  
 $S_f$  number of single tags for flavor modes.  
 $K'_i(K'_{-i})$ , yields in each bin of Dalitz plot in flavor modes.

From the Double Dalitz modes, we are able to find  $c_i, c'_i, s_i, s'_i$

$$M'_{i,j} = \frac{N_{D,\bar{D}}}{2S_f^2} \left( K_i K'_{-j} + K_{-i} K'_j + 2 \sqrt{K_i K'_{-j} K_{-i} K'_j} (c_i c'_j + s_i s'_j) \right)$$

$i^{\text{th}}$  bin for  $K_S^0 \pi^+ \pi^-$   
 $j^{\text{th}}$  bin for  $K_L^0 \pi^+ \pi^-$

$M_{i,j}$  yields in bin  $i$  of  $K_S^0 \pi^+ \pi^-$  Dalitz plot and bin  $j$  of  $K_L^0 \pi^+ \pi^-$  Dalitz plot.  
 $S_f$  number of single tags for flavor modes.  
 $N_{D,\bar{D}}$  total number of  $D^0 \bar{D}^0$  events.  
 $K_i(K_{-i})$ , yields in each bin of  $K_S^0 \pi^+ \pi^-$  Dalitz plot in flavor modes.  
 $K'_j(K'_{-j})$ , yields in each bin of  $K_L^0 \pi^+ \pi^-$  Dalitz plot in flavor modes.